EPS 7th Grade Mathematics Instructional Sequence 2010-11

Unit	Start date:	**7th Grade Core Content Standard**
Stretching and Shrinking	Sept 8	**7.2/7.3** *Proportionality and Similarity*: solving problems involving similar figures (geometric) and problems relating to scale.
Comparing and Scaling	Oct 18	**7.2** *Proportionality and Similarity*: solving single and multi-step proportion problems, including measurement conversions
Accentuate the Negative	Nov 18	**7.1** *Rational Numbers*: $+ - \times \div$ rational numbers (fractions, decimals, integers) including both positive and negative numbers.
Moving Straight Ahead	Jan 18	**7.1/7.2** *Linear Equations and Proportionality*: including writing and solving equations, graphing linear relationships.
Filling & Wrapping	Mar 3	**7.3** *Surface Area and Volume*: All aspects of three-dimensional figures, including problem solving.
Data Distributions	Apr 18	**7.4** *Probability and <u>Data</u>*: includes statistical measures of central tendencies, variability, various data plots.
What Do You Expect	May 31	**7.4** *<u>Probability</u> and Data*: sample spaces, theoretical probability and experimental outcomes, how to determine probability.
Throughout each Unit	All year	**7.5** *Additional Content*: coordinate graphs, exponents are woven into the units when appropriate. **7.6** *Reasoning, Problem Solving, and Communication*: Throughout the year, in various units, multiple representations – numbers, words, pictures (graphs), and symbol - are used to communicate mathematical ideas. Problem solving includes the algebra of proportionality and the use of rational numbers.

Grade 7

Adopted Washington State

Mathematics Standards

April 28, 2008

7.1. Core Content: Rational numbers and linear equations (Numbers, Operations, Algebra)

Students add, subtract, multiply, and divide rational numbers—fractions, decimals, and integers—including both positive and negative numbers. With the inclusion of negative numbers, students can move more deeply into algebraic content that involves the full set of rational numbers. They also develop the algebraic skill of solving equations that require more than one step, allowing them to approach problems that deal with a wider range of contexts than before. Using generalized algebraic skills and approaches, students can pursue a wide range of problems involving any type of rational number, adapting strategies for solving one problem to different problems in different settings with underlying similarities.

7.2. Core Content: Proportionality and similarity (Operations, Geometry/Measurement, Algebra)

Students extend their work with ratios to solve problems involving a variety of proportional relationships, such as making conversions between measurement units or finding the percent increase or decrease. They also solve problems involving the proportional relationships related to similar figures, and in so doing reinforce an important connection between numerical operations and geometric relationships. Students graph proportional relationships and identify the rate of change as the slope of the related line. The skills and concepts related to proportionality represent some of the most important connecting ideas across K–12 mathematics. With a good understanding of how things grow proportionally, students can understand the linear relationships that are the basis for high school mathematics. If learned well, proportionality can open the door for success in much of secondary mathematics.

7.3. Core Content: Surface area and volume (Geometry/ Measurement)

Students extend their understanding of area and perimeter to finding the surface area and volume of three-dimensional figures. They apply formulas and solve a range of problems involving three-dimensional objects, including problems people encounter in everyday life, in certain types of work, and in other school subjects. With a strong understanding of how to work with both two-dimensional and three-dimensional figures, students have an important base for the geometry they will study in high school.

7.4. Core Content: Probability and data (Data/Statistics/Probability)

Students apply their understanding of rational numbers and proportionality to concepts of probability. They begin to understand how probability is determined, and they make related predictions. Students revisit how to interpret data, now using sophisticated types of data graphs and thinking about the meaning of certain statistical measures. Statistics, including probability, is considered one of the most important and practical fields of study for making sense of quantitative information, and it plays an important part in secondary mathematics in the 21st century.

7.5. Additional Key Content (Numbers, Algebra)

Students extend their coordinate graphing skills to plotting points with both positive and negative coordinates on the coordinate plane. Using pairs of numbers to locate points is a necessary skill for reading maps and tables and a critical foundation for high school mathematics. Students further prepare for algebra by learning how to use exponents to write numbers in terms of their most basic (prime) factors.

7.6. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade seven, students solve problems that involve positive and negative numbers and often involve proportional relationships. As students solve these types of problems, they build a strong foundation for the study of linear functions that will come in grade eight.

Mathematics content based on
Adopted Washington State
K-8 Mathematics Standards,
April 28, 2008,
Office of the Superintendent of Public Instruction

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Grade Seven Performance Expectations

Rational Numbers and Linear Equations (numbers, operations, algebra)

7.1.A **Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.** (6.5.C)

7.1.B *Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.* (6.1.B)

7.1.C *Fluently and* **accurately add, subtract, multiply, and divide rational numbers.** (6.1.D) (6.1.F)

7.1.D *Define and* **determine the absolute value of a number.**

7.1.E **Solve two-step linear equations.** (6.2.E) (8.1.A)

7.1.F **Write an equation that corresponds to a given problem situation,** *and describe a problem situation that corresponds to a given equation.* (6.2.A)

7.1.G **Solve single- and multi-step word problems involving rational numbers and verify the solutions.** (6.1.H)

Proportionality and Similarity (operations, geometry/measurement, algebra)

7.2.A *Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.* (6.5.A)

7.2.B **Solve single- and multi-step problems involving proportional relationships and verify the solutions.** (6.4.C)

The performance expectation identified in the parentheses represents a connection to a previous or future grade level performance expectation.

Bold and *italicized* formatting based on the most current version of the MSP Mathematics Item Specifications. Expectations for the state assessment are in **bold text**. Expectations for local instruction and assessment appear in *italicized* text.
** This performance expectation may be included in items assessing process performance expectaions.

7.2.C **Describe proportional relationships in similar figures and solve problems involving similar figures.**

7.2.D *Make scale drawings and* **solve problems related to scale.**

7.2.E **Represent proportional relationships using graphs, tables, and equations, and make connections among the representations.**

7.2.F **Determine the slope of a line corresponding to the graph of a proportional relationship** *and relate slope to similar triangles.* (5.4.D)

7.2.G **Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.** **

7.2.H *Determine whether or not a relationship is proportional and explain your reasoning.*

7.2.I **Solve single- and multi-step problems involving conversions within or between measurement systems and verify the solutions.**

Surface Area and Volume (geometry/measurement)

7.3.A **Determine the surface area and volume of cylinders using the appropriate formulas** *and explain why the formulas work.* (6.4.A) (6.4.E)

7.3.B **Determine the volume of pyramids and cones using formulas.** (6.4.F)

7.3.C **Describe the effect that a change in scale factor on one attribute of a two- or three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.**

7.3.D **Solve single- and multi-step word problems involving surface area or volume and verify the solutions.** (6.4.C)

Probability and Data (data/statistics/probability)

7.4.A **Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.** (6.3.G)

7.4.B **Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.** (6.3.G) (8.3.F)

7.4.C **Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range)** *and evaluate the suitability and limitations of using each measure for different situations.* (5.5.B) (8.3.A)

7.4.D **Construct and interpret histograms, stem-and-leaf plots, and circle graphs.** (8.3.B)

7.4.E **Evaluate different displays of the same data for effectiveness and bias, and explain reasoning.** ** (8.3.D) (8.3.E)

Additional Key Content (numbers, algebra)

7.5.A **Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.** (5.4.D)

7.5.B **Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.** (5.2.D) (5.5.A)

Reasoning, Problem Solving, and Communication

7.6.A *Analyze a problem situation to determine the question(s) to be answered.* (6.6.A) (8.5.A)

7.6.B **Identify relevant, missing, and extraneous information related to the solution to a problem.** (6.6.B) (8.5.B)

7.6.C *Analyze and compare mathematical strategies for solving problems, and* **select and use one or more strategies to solve a problem.** (6.6.C) (8.5.C)

7.6.D **Represent a problem situation,** *describe the process used to solve the problem, and* **verify the reasonableness of the solution.** (6.6.D) (8.5.D)

7.6.E *Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.* (6.6.E) (8.5.E)

7.6.F *Apply a previously used problem-solving strategy in a new context.* (6.6.F) (8.5.F)

7.6.G **Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.** (6.6.G) (8.5.G)

7.6.H *Make and test conjectures based on data (or information) collected from explorations and experiments.* (6.6.H) (8.5.H)

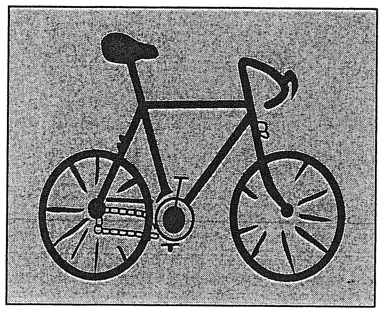
Stretching & Shrinking (CMP2)

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
Investigation 1 Enlarging and reducing shapes: Do 1.1 (1 day) , 1.2 (1 day) , 1.3 (1 day) and mathematical reflections (1/2 DAY)	3 ½			**7.2.C Describe proportional relationships in similar figures and solve problems involving similar figures.** **7.2.D** *Make scale drawings and* **solve problems related to scale.** **7.3.C Describe the effect that a change in scale factor on one attribute of a two-or-three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.** **7.5.A Graph ordered pairs of rational numbers and determine the coordinates, of a given point in the coordinate plane.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Investigation 2 Similar Figure: Do 2.1 (2 days) Must do launch to practice x and y coordinate p. 33, 2.2 (1 day) , 2.3 (1 day) and mathematical reflections (½ day) Differentiate use of "similar" in everyday language vs **"mathematically similar"**	4 1/2			
Check up 1 (answers in binder)	½	Binder/CMP2 teacher express grade 7		
Investigation 3 Similar Polygons: Do 3.1 (1 day), 3.2 (1 day), 3.3 (1 day) and mathematical reflections (½ day)	3 1/2			
Partner quiz (see answers with Check up 1 answers)	1	Binder/CMP2 teacher express grade 7		
Investigation 4 Similarity and Ratios: Do 4.1 (1 day), 4.2 (1 day), 4.3 (1 day) and mathematical reflections (1/2 day)	3 1/2			
For additional practice: **Ratios and Similar Rectangles Worksheet 3-1,2,3** with teacher instructions p. 35-40(from MGMP Similarity & Equivalent Fractions) OR **INV 4 Skill Practice: Similarity and Ratios p34**	1	Both in binder		
Check up 2 (see answers with check up 1 answers)	1/2	Binder/CMP2 teacher express grade 7		
Investigation 5 Using Similar Triangles and Rectangles: Do 5.1 (1 day), 5.2 (1 day), 5.3 (1 day), mathematical reflections (1/2 day)	3 1/2			
Looking Back and Looking Ahead, p. 98	1		Optional	
District Unit Assessment	1			
Review & Reflect Assessment Student Self-Assessment	1			
Total Instructional Days for Stretching & Shrinking:	25 total			

Contents in Stretching and Shrinking

Check-Up 1

1. The coach took a digital photo of the new cycling team bike. She sent an 8 cm by 10 cm photo to each team member.



a. If the photo were enlarged by a scale factor of 150% of its original size, what would be the new length and width?

b. Imagine you want to make a 2 cm by 2.5 cm copy of the original photo. What percent should you use?

c. What scale factor relates the side lengths in the original photo to those in the smaller photo?

d. How will the angles in the original photo compare to the corresponding angles in the smaller photo?

e. How will the perimeter of the original photo compare to that of the smaller photo?

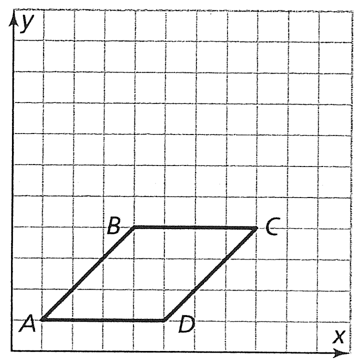
f. Find the areas of the original and the smaller photos. How do these areas compare?

Name ______________________________ Date ____________ Class ____________

Check-Up 1 *(continued)*

for use after **Investigation 1**

2. While designing a video game, Victor drew parallelogram $ABCD$ with vertex coordinates (1,1), (4,4), (8,4) and (5,1) like the one on the coordinate grid below.



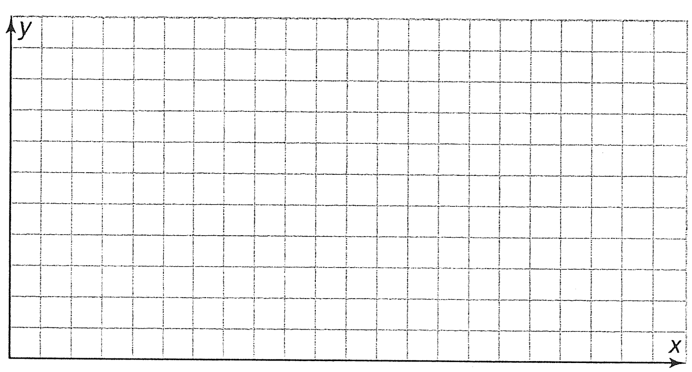
a. Find the area and perimeter of parallelogram $ABCD$.

Area = Perimeter =

b. Draw another rectangle $EFGH$ with vertex coordinates related to parallelogram $ABCD$ by the rule $(0.5x, 0.5y)$.

Point	(x, y)
A	(1, 1)
B	(4, 4)
C	(8, 4)
D	(5, 1)

Point	(0.5x, 0.5y)
E	
F	
G	
H	



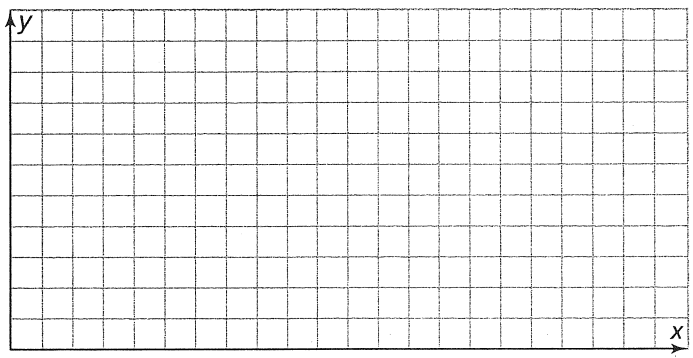
c. Find the area and perimeter of parallelogram $EFGH$.

Area = Perimeter =

d. Draw another parallelogram *JKLM* with vertex coordinates related to parallelogram *ABCD* by the rule $(2x, 1.5y)$.

Point	(x, y)
A	(1, 1)
B	(4, 4)
C	(8, 4)
D	(5, 1)

Point	(2x, 1.5y)
J	
K	
L	
M	



e. Find the area and perimeter of parallelogram *JKLM*.

Area = Perimeter =

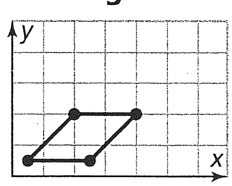
f. Which parallelogram, *EFGH* or *JKLM*, is similar to parallelogram *ABCD*? Justify your answer.

Check-Up 1

1. a. 12 cm by 15 cm
   b. 25% c. $\frac{1}{4}$
   d. The corresponding angles in both photos will have the same measures.
   e. Perimeter of original = 36 cm
   Perimeter of reduced = 9 cm
   The perimeter of the original is 4 times the size of the reduced photo. OR
   The perimeter of the reduced photo will be $\frac{1}{4}$ the perimeter of the original photo.
   f. Area of original = 80 cm^2.
   Area of reduced = 5 cm^2.
   The area of the original photo is 16 times as large as the area of the reduced photo. OR
   The area of the reduced photo will be $\frac{1}{16}$ the area of the original photo.
2. a. Area = 12 square units
   Perimeter ≈ 16 units
   b.

Point	(0.5*x*, 0.5*y*)
E	(0.5, 0.5)
F	(2, 2)
G	(4, 2)
H	(2.5, 0.5)

Parallelogram *EFGH*



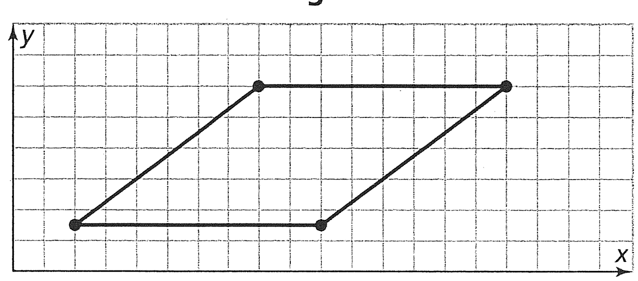
   c. Area = 3 square units
   Perimeter ≈ 8 units
   d. (Figure 1)
   e. Area = 36 square units
   Perimeter ≈ 31 units
   f. *EFGH* is similar to *ABCD*, because the corresponding sides are related by the same scale factor, $\frac{1}{2}$. Also, corresponding angles appear to be equal.

Check-Up 2

1. Rectangle 1 is similar to Rectangle 2. The ratio of length to width is 3 to 2. Corresponding sides in Rectangles 1 and 2 are related by a scale factor of 1.5. The corresponding angles in Rectangles 1 and 2 have the same measure (all 90°).
2. a. side AC = 20, side AB = 16
   b. 4
   c. $\frac{1}{4}$
   d. The perimeter of ABC is 4 times the perimeter of triangle DEF.
   e. The area of triangle ABC is 16 times the area of triangle DEF.
3. a. side TR = 10 units
   b. side XZ = 26 units
   c. angle T = 110°
   d. angle Z = 20°

Figure 1

Parallelogram JKLM



Point	(2*x*, 1.5*y*)
J	(2, 1.5)
K	(8, 6)
L	(16, 6)
M	(10, 1.5)

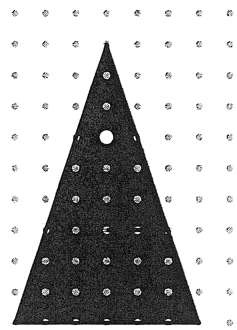
Name ____ Date ____________ Class ____________

Partner Quiz

for use after **Investigation 3**

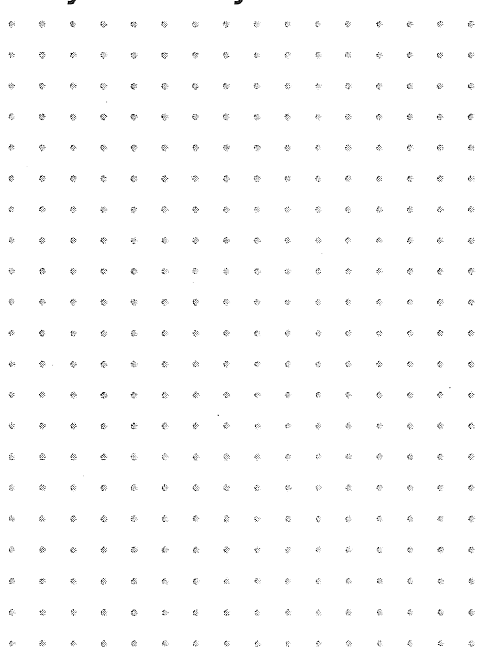
Stretching and Shrinking

1. Ryan drew a one-eyed triangle character on dot paper. Ashley used the rule $(3x, 3y)$ to enlarge Ryan's drawing and she drew the character below.

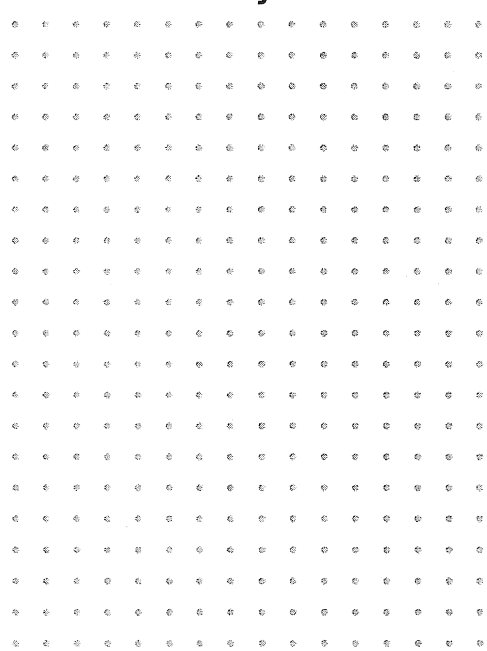


a. Simone saw Ashley's drawing and doubled all the lengths to create her own character. On the grids below, sketch Ryan's original character and Simone's new version of Ashley's character.

Ryan's One-eyed Character



Simone's One-eyed Character



b. Are Ryan's and Simone's characters similar? Explain.

c. Write a rule that would create Simone's character from Ryan's character.

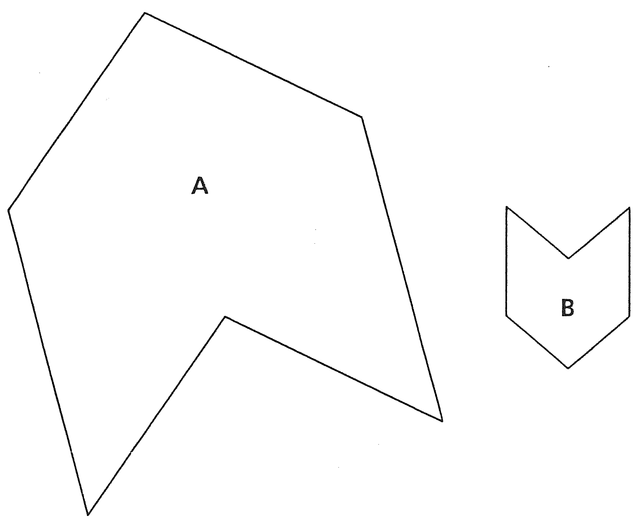
2. Megan wanted to make a new video game character.

a. Write a rule that would transform Mug (x, y) (see Problem/Labsheet 2.1) into Slug who is very wide and not very tall.

b. Megan wanted Slug to move up (but not over) on the grid. What rule could do this for her?

c. Is Slug similar to Mug? Why or why not?

3. Are shapes A and B similar? Explain your answer.



Partner Quiz

1. a. (Figure 2)

   b. Ryan's and Simone's characters are similar: the shapes are the same, corresponding angles are the same measure, and the corresponding sides grow by the same scale factor. The scale factor from Ryan's character to Simone's character is 6.
   (Note: If students use the transitive property to answer this question—since Ryan's character is similar to Ashley's, and Ashley's is similar to Simone's, then Ryan's is similar to Simone's—you may want to ask them to support the answer with an explanation based on the properties of similar figures.)

   c. $(6x, 6y)$

2. a. Any rule in which the coefficient of x is relatively large compared to the coefficient of y will work; for example, $(5x, 2y)$ or $(3x, y)$ or $(10x, 0.5y)$

   b. Student should add some positive number to the second coordinate in their rule from part a; for example, $(5x, 2y + 4)$ or $(3x, y + 3)$ or $(10x, 0.5y + 8)$

   c. Slug is not similar to Mug, because Slug is stretched more horizontally than vertically. The figures have different shapes, different angles, and no consistent scale factor.

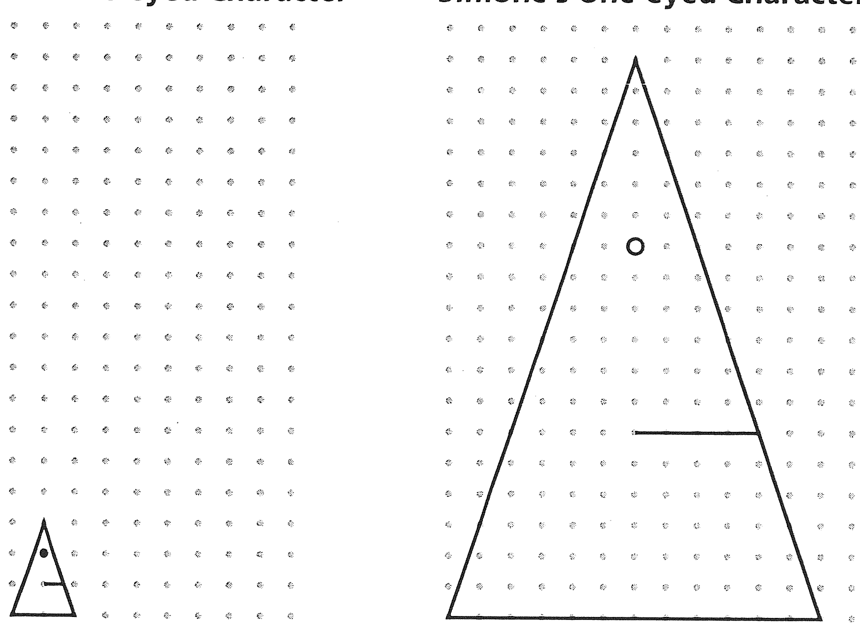
3. Shapes A and B are similar. They have the same basic shape, corresponding angles are equal, and the side lengths of shape A are about 3 times the corresponding side lengths of B. (Note: Students may determine this by measuring, tracing, or cutting out the two shapes to compare their angles and sides.)

Multiple Choice Items

1. B 2. F 3. C 4. F

Figure 2

Ryan's One-eyed Character **Simone's One-eyed Character**



TEACHER ACTION	TEACHER TALK	EXPECTED RESPONSE
	In similar rectangles the ratios $\frac{\text{short side}}{\text{long side}}$ are equivalent fractions.	
	Which of these noses are similar?	1, 2, and 3.
	We can also use this as a test for similar rectangles. If the ratios are equal, then the rectangles are similar.	
	We said earlier that similar figures must also have angle measure preserved. Why don't we need to check angles on rectangles?	All the angles of rectangles are 90°.
Students need Worksheet 3-1, Which Rectangles Are Similar? and Worksheet 3-2, Similar Rectangles, and grid paper. Put a transparency of Worksheet 3-1, Which Rectangles Are Similar? on the overhead.	All of these figures are rectangles. Your task is to find which ones are similar.	
Ask.	How could we do this?	Look for one with the same shape. (If this response is given, say it is not always easy to use "same shape" as a test. Rectangles all look much alike. Ask how to be sure the shape is the same.)
	How else could we do it?	Find the ratios of short side to long side. If the ratios are equal, then the rectangles are similar.
	Could we use the ratio of long side to short side?	Yes, just as long as we are consistent.

TEAC ACTION	TEACHER TALK	EXPECTED RESPONSE
Use a one-fourth page transparency of grid paper (Materials 3-1).	In this activity you will collect some other information about each rectangle, its perimeter and its area.	
Demonstrate how to use the grid as a ruler to find the lengths and areas. Place the grid over the figure and count squares for area and units for length.	What is the perimeter of a rectangle?	The distance around.
	How do we find the perimeter using the grid?	Count the lengths on the transparency.
	How do we find the area?	Count all the squares in the rectangle or multiply the two sides together: $A = W \times L$ or $A =$ long side $\times$ short side.

Fill in the first row of the chart on Worksheet 3-2.

		Short Side a	Long Side b	Ratio $\frac{a}{b}$	Area	Perimeter
Rectangles	1	3 units	4 units	$\frac{3}{4}$	12 Square units	14 units

Give students time to collect data and fill in the chart.

OBSERVATIONS	POSSIBLE RESPONSES
Some students will make the mistake of trying to count a row of squares when measuring perimeter.	Use the language of walking around the block or wrapping a string around the rectangle.
Some students confuse area and perimeter.	Stress the basic difference in units of measurement for length and area. Be sure groups of students are focusing on "distance around" and "covering" as they measure the rectangles. Ask a question such as, "George measured a rectangle and got 12. What could that . . . mean?"

Activity 3 *Summarize*

TEACHER ACTION | TEACHER TALK | EXPECTED RESPONSE

Put a transparency of Worksheet 3-2, Similar Rectangles, on the overhead and collect the data from the class for rectangles.

Rectangles	Short side a	Long side b	Ratio $\frac{a}{b}$	Area	Perimeter
1	3 units	4 units	$\frac{3}{4}$	12 sq. units	14 units
2	6	8	$\frac{6}{8} = \frac{3}{4}$	48	28
3	9	12	$\frac{9}{12} = \frac{3}{4}$	108	42
4	12	16	$\frac{12}{16} = \frac{3}{4}$	192	56
5	15	20	$\frac{15}{20} = \frac{3}{4}$	300	70
6	10	14	$\frac{10}{14} = \frac{5}{7}$	140	48

Ask.

Which rectangles are similar?

Rectangles 1, 2, 3, 4, and 5.

Could we have decided this by looking at just the shape? Why?

No; same shape is ambiguous, hard to tell.

How did you decide which rectangles are similar?

Check the ratio of the sides: $\frac{\text{short side}}{\text{long side}}$.

We found the ratio $\frac{a}{b}$ or the short side to the long side. Could we use $\frac{b}{a}$?

Yes we could use $\frac{b}{a}$ provided we use it for all rectangles. $\frac{b}{a}$ is an improper fraction; $\frac{b}{a}$ is larger than 1.

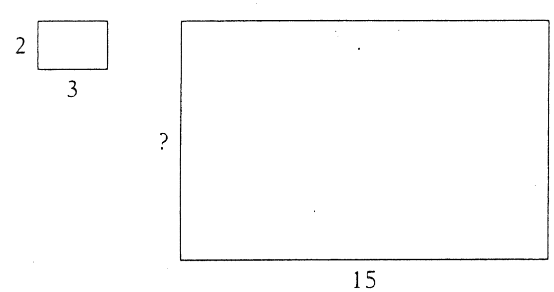
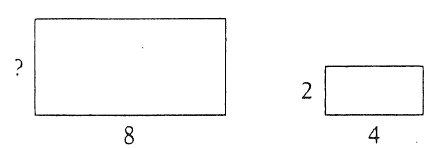
What happens?

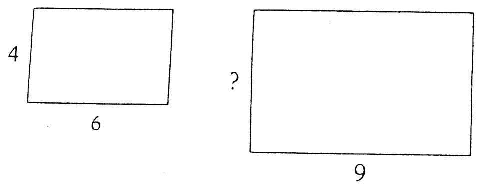
$\frac{a}{b} = \frac{3}{4}$

$\frac{b}{a} = \frac{4}{3}$

Why do we not have to check angles?

Because rectangles have all angles equal to 90°.

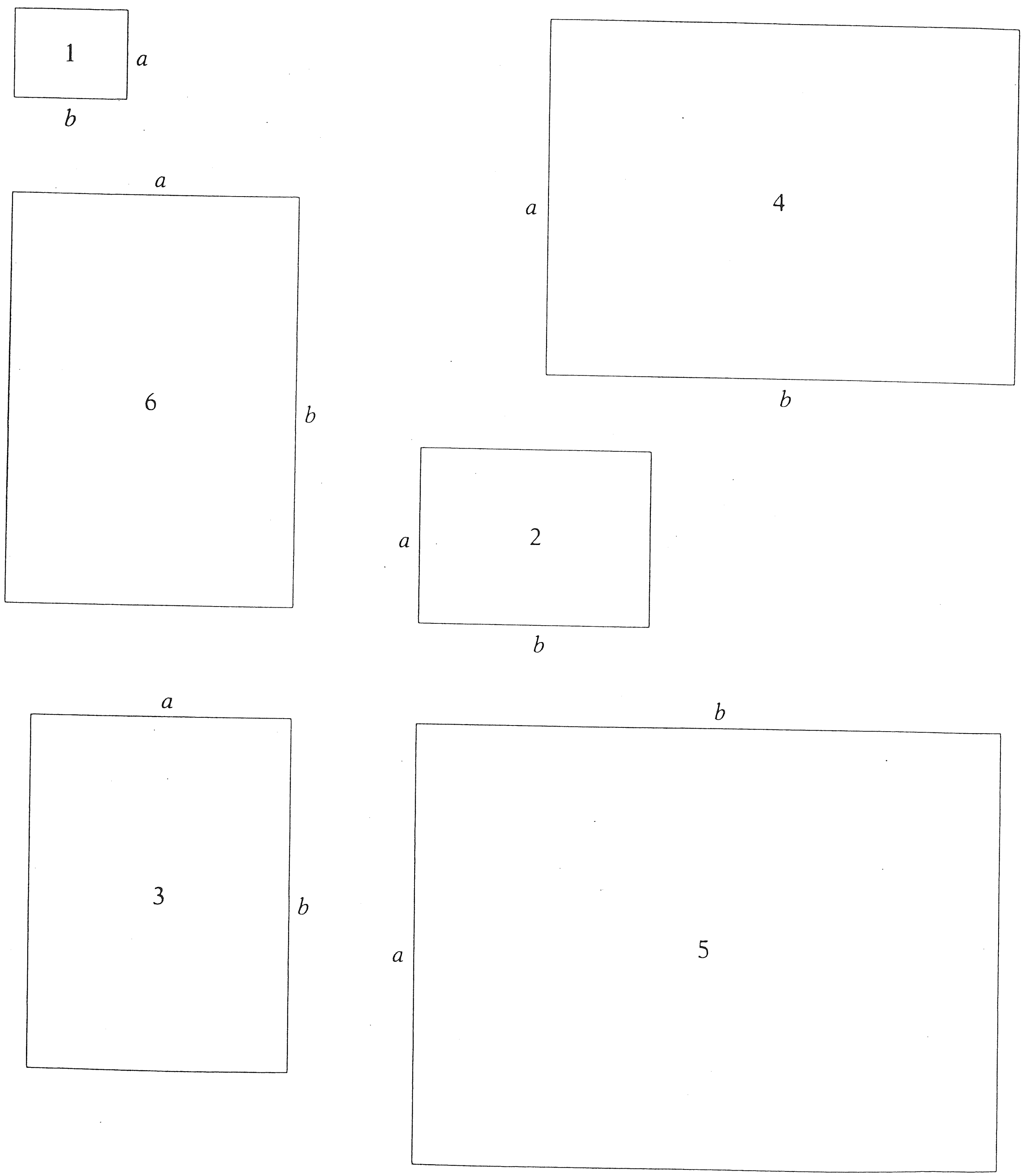
TEACHER ACTION	TEACHER TALK	EXPECTED RESPONSE
Ask.	Let's put some more similar rectangles on the list.	
Enter the new rectangles.	If the short side is 30, what's the long side of the similar rectangle?	40; $\frac{30}{40} = \frac{3}{4}$.
	If the long side is 32, what's the short side?	24
	What are some other rectangles that are similar to the rectangles with ratio of sides $\frac{3}{4}$?	Various answers.
	Let's look at another pair of similar rectangles.	
Put the following rectangles on the board or overhead. 	What is the missing side?	10
	What do you multiply the smaller rectangle's edges by to get the larger rectangle? This is called the *scale factor* from the smaller to the larger rectangle. It tells us what to multiply the dimensions of the original by to get the new figure.	5
Show these. 	What is the missing side?	4
	What's the perimeter of the larger rectangle?	24
	What's the perimeter of the smaller rectangle?	12
	What is the *scale factor* from the larger to the smaller rectangle? This means that we multiply the larger (the original) dimensions by $\frac{1}{2}$ to get the dimensions of the new figure.	$\frac{1}{2}$

TEACHER ACTION	TEACHER TALK	EXPECTED RESPONSE
	What about these? If the figures are similar, what is the missing side?	6 units. If students have difficulty, write the equivalent fractions $\frac{4}{6} = \frac{?}{9}$ and suggest they rewrite $\frac{4}{6}$ in lowest terms.
If students are familiar with finding equal fractions, this part should go quickly. Otherwise this is a good time to review or reinforce equivalent fractions: renaming to lower terms or to higher terms.	When the figures are given to be similar, what can we assume about corresponding angles?	They are equal.
	About the ratios of corresponding sides?	They are equal.
Distribute Worksheet 3-3, Ratios and Similar Rectangles. (You may prefer to assign this as homework.)	Here are some practice problems that use the ideas about similarity that we have studied.	

NAME ______________________

Which Rectangles Are Similar?

Use your grid to measure each rectangle. Record the information on Worksheet 3-2, Similar Rectangles. Determine which of the rectangles are similar.



$\frac{1}{2}$-cm Grid

NAME ____________________

Similar Rectangles

Rectangle	Short side a	Long side b	Ratio $\frac{a}{b}$	Area	Perimeter
1					
2					
3					
4					
5					
6					

Which rectangles are similar? ____________________

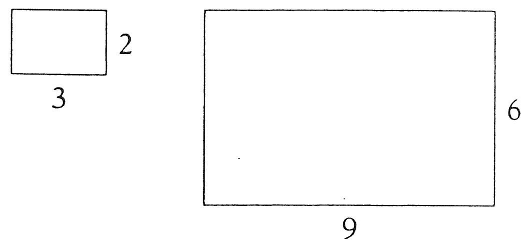
Give a rule for testing rectangles to see if they are similar. ____________________

NAME ______________________

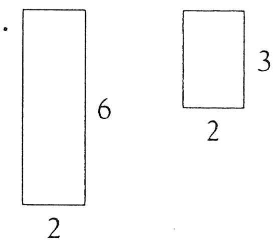
Ratios and Similar Rectangles

Without measuring, which pairs of rectangles (1–4) are similar? ______________________

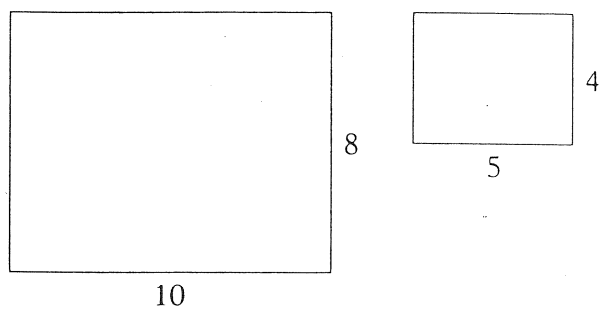
1.



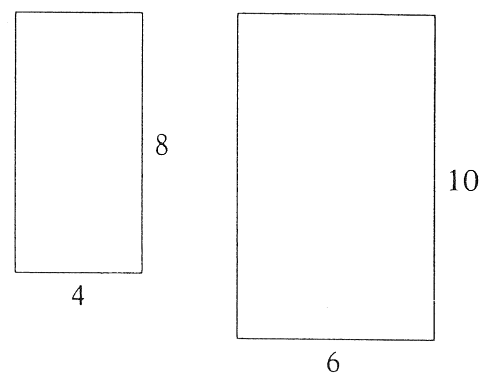
2.



3.

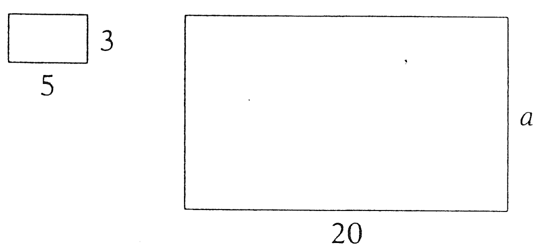


4.

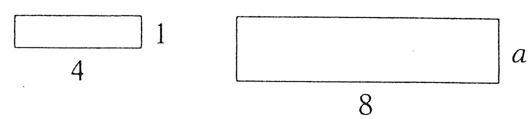


Each pair of rectangles (5–10) are similar. Find the missing measurements.

5. Side a = ______

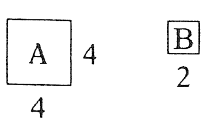


6. Side a = ______



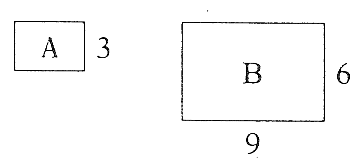
7. Perimeter A = ______

Perimeter B = ______



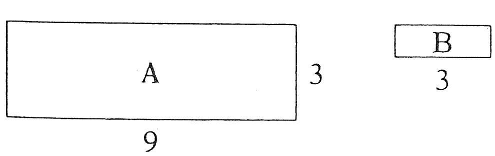
8. Perimeter A = ______

Perimeter B = ______



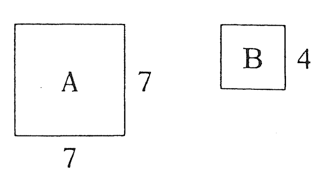
9. Perimeter A = ______

Perimeter B = ______



10. Area A = ______

Area B = ______



NAME

Similar Rectangles

Rectangle	Short side a	Long side b	Ratio $\frac{a}{b}$	Area	Perimeter
1	3 units	4 units	$\frac{3}{4}$	12 sq units	14 units
2	6	8	$\frac{6}{8}$ or $\frac{3}{4}$	48	28
3	9	12	$\frac{9}{12}$ or $\frac{3}{4}$	108	42
4	12	16	$\frac{12}{16}$ or $\frac{3}{4}$	192	56
5	15	20	$\frac{15}{20}$ or $\frac{3}{4}$	300	70
6	10	14	$\frac{10}{14}$ or $\frac{5}{7}$	140	48

Which rectangles are similar? 1, 2, 3, 4, 5

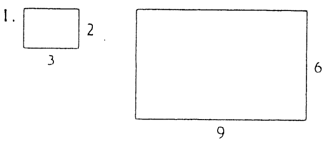
Give a rule for testing rectangles to see if they are similar.

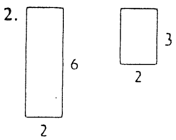
Ratio of short side to long side is the same.

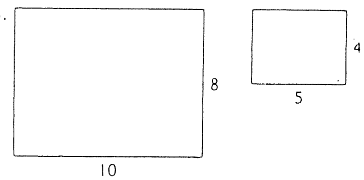
NAME

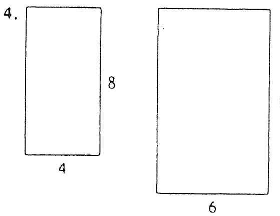
Ratios and Similar Rectangles

Without measuring, which pairs of rectangles (1–4) are similar? 1, 3

1.


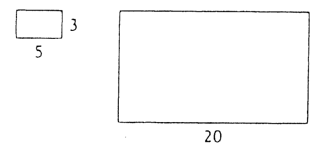
2.


3.


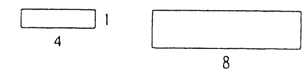
4.


Each pair of rectangles (5–10) are similar. Find the missing measurements.

5. Side a = 12

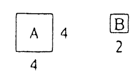


6. Side a = 2



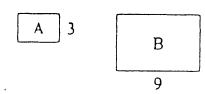
7. Perimeter A = 16

Perimeter B = 8



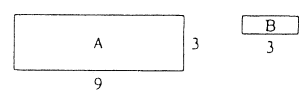
8. Perimeter A = 15

Perimeter B = 30



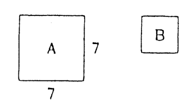
9. Perimeter A = 24

Perimeter B = 8



10. Area A = 49

Area B = 16



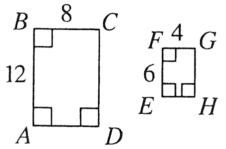
Name ______________________________ Date ____________ Class ____________

Skill: Similarity and Ratios

Investigation 4

Stretching and Shrinking

Tell whether each pair of polygons is similar. Explain why or why not.

1.


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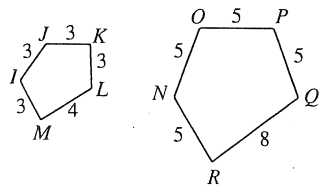
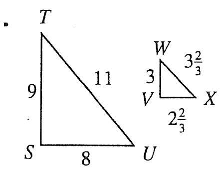
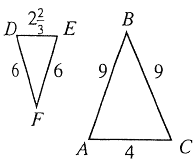
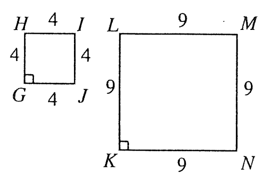
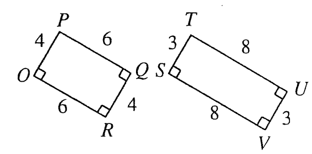


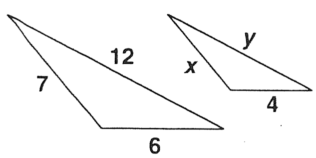
3.


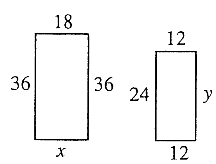
4.


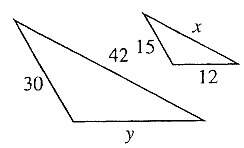
5.


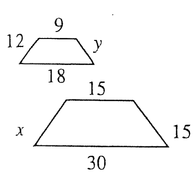
6.


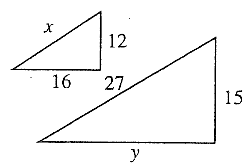
Exercise 7–11 show pairs of similar polygons. Find the missing side lengths.

7.


8.


9.


10.


11.


Stretching and Shrinking Answers

Investigation 4 Skills Practice

Skill: Similarity and Ratios

1. yes; $ABCD \sim EFGH$
2. no
3. yes; $\triangle STU \sim \triangle VWX$
4. yes; $\triangle DEF \sim \triangle CAB$
5. yes; $GHIJ \sim KLMN$

6. no
7. $x = 4\frac{2}{3}$; $y = 8$
8. $x = 18$; $y = 24$
9. $x = 21$; $y = 24$
10. $x = 20$; $y = 9$
11. $x = 21\frac{3}{5}$; $y = 20$

Comparing & Scaling (CMP2)

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	**6-8 Performance Expectations /Additional Targets**
Additional Practice from Inv 1-3 (these are taught in 6th) **Review Unit Rates: Inv 3 Skill Sheet- Finding and Using Unit Rates p52 -53**	2	Binder/CMP2 Disc		*7.2.A Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.* **7.2.B Solve single- and –multi step problems involving proportional relationships and verify the solutions** *7.2.H Determine whether or not a relationship is proportional and explain your reasoning.* **7.2.I Solve single-and multi-step problems involving conversions within or between measurement systems and verify the solutions.**
Problem 4.1, Setting Up and Solving Proportions, p. 48	2			
Problem 4.2, Everyday Use of Proportions, p. 52	1			
Problem 4.3, Developing Strategies for Solving Problems, p. 53	1			
Measuring Up (includes conversion activities) Lesson 1 Measurement Terms, Lesson 4 Do you Measure up?, Lesson 7 Off the Scale— Illuminations Online	3	Online/binder		
Topic 5: Dimensional Analysis	1	Online lesson		
European Shopping Adventure Interactive (if you have computer access) or *Enrichment 5- "Using Rates and proportional Reasoning"* in addtl prac page	1	Online and binder		
Big Math and Fries activity – McDonald's Nutrition Facts in binder or http://nutrition.mcdonalds.com/nutritionexchange/nutritionfacts.pdf	2	Online and binder		
***CMP2* Mathematical Reflections, Investigation 4, p. 62**	1			
Looking Back and Looking Ahead, p. 84 – 85, questions 1, 2, 4, and Explain Your Reasoning	2			
Comparing & Scaling Unit Assessment	2			
Review & Reflect Assessment, Student Self-Assessment	1			Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Total Instructional Days for Comparing & Scaling:	19			

Contents in Comparing and Scaling

Name ______________________ Date ___________ Class ___________

Skill: Finding and Using Rates

Write the unit rate for each situation.

1. travel 250 miles in 5 hours
2. earn $75.20 in 8 hours
3. read 80 pages in 2 hours
4. type 8,580 words in 2 hours 45 minutes
5. manufacture 2,488 parts in 8 hours
6. 50 copies of a book on 2 shelves
7. $30 for 6 books
8. 24 points in 3 games

Find each unit price. Then determine the better buy.

9. paper: 100 sheets for $0.99
   500 sheets for $4.29
10. peanuts: 1 pound for $1.29
    12 ounces for $0.95
11. crackers: 15 ounces for $1.79
    12 ounces for $1.49
12. apples: 3 pounds for $1.89
    5 pounds for $2.49
13. mechanical pencils: 4 for $1.25
    25 for $5.69
14. bagels: 4 for $0.89
    6 for $1.39

Name ______________________ Date __________ Class __________

Additional Practice

15. a. Yolanda and Yoko ran in a 100-yd dash. When Yolanda crossed the finish line in 15 seconds, Yoko was 10 yards behind her. The girls then repeated the race, with Yolanda starting 10 yards behind the starting line. If each girl ran at the same rate as before, who won the race? By how many yards?

b. Assume the girls run at the same rate as before. How far behind the starting line should Yolanda be in order for the two to finish in a tie?

16. During the breaststroke competitions of a recent Olympics, Nelson Diebel swam 100 meters in 62 seconds, and Mike Bowerman swam 200 meters in 130 seconds. Who swam at a faster rate?

17. During a vacation, the Vasquez family traveled 174 miles in 3 hours on Monday, and 290 miles in 5 hours on Tuesday. Write an equation relating miles m traveled to hours h.

Name ______________________________ Period ________

Comparing and Scaling: Problem Solving Using Unit Rates

Find each unit rate.

1. $\frac{60 \text{ miles}}{2 \text{ hours}}$	**2.** $\frac{96 \text{ words}}{4 \text{ minutes}}$	**3.** $\frac{300 \text{ miles}}{15 \text{ gallons}}$
4. $\frac{12 \text{ days}}{2 \text{ jobs}}$	**5.** $\frac{\$3.30}{3 \text{ pens}}$	**6.** $\frac{24 \text{ ounces}}{1.5 \text{ servings}}$
7. $\frac{32 \text{ pounds}}{8 \text{ inches}}$	**8.** $\frac{30 \text{ feet}}{3 \text{ seconds}}$	**9.** $\frac{15 \text{ kilometers}}{5 \text{ hours}}$

11. José spent $36 for 4 movie tickets. Find the price per ticket.

12. Rob spent $3.30 for 6 large cookies. Find the price per cookie.

13. Polly used 10 gallons of gas to drive 235 miles on a trip. Find how many miles per gallon Polly's car got on the trip.

14. Tran rode his scooter 10 miles in 1.5 hours. Find how many miles per hour he rode.

15. Maria could buy 6 songs online for $3.00 at Songs-R-Us, or she could pay $0.45 per song at Music Hooray.

a. Find the unit rate of dollars per song for Songs-R-Us.

b. Which company charges less per song?

16. Luke walked 2 miles in 40 minutes. He determined his unit rate was 20 minutes per mile. Sally informed him his rate was 0.05 miles per minute. Are these both accurate unit rates? Explain.

Measuring Up

Measurement Terms

This lesson introduces relationships between measurement and geometry. The activities build on students' prior knowledge as students work with partners and as a whole class to identify and classify terms to develop their understanding of measurement.

Learning Objectives

Students will

- identify and classify terms related to measurement
- understand the relationships between terms of measurement

Materials

Chart paper

Markers
Index cards

Instructional Plan

Have students brainstorm a list of all the terms they know that relate to measurement. Record their answers in list form on a chart. Students may also write each term on a separate index card.

Student answers may include length, weight, height, meter, centimeter, liter, mile, inch, foot, and so forth.

Organize the students in pairs and have them group and label the terms that the class has just brainstormed. This helps students establish connections among the various categories of terms. Students can move the cards around on the chart paper as they group the terms.

Have the class reach a consensus on the major categories in which the terms can be grouped, and record these categories on a chart. Then ask students to group terms that have common attributes. For example, students may place the terms liter, meter, or gram in a "metric" category. Or, they may categorize the terms according to length, weight, volume, etc. A sample table appears below.

Sample Measurement Terms

Length	Volume (Capacity)	Weight	Mass	Time	Temperature
inch	gallon	pound	milligram	second	degrees Celsius
foot	quart	ounce	gram	minute	degrees Fahrenheit

yard	pint	ton	kilogram	hour	
mile	cup			day	
meter	fluid ounce			month	
centimeter	liter			year	
kilometer	milliliter			decade	
millimeter	kiloliter			century	

Students may work in groups or individually to write brief sentences about what they know about the terms.

This activity allows you to get an idea of what the students know before you delve further into the concepts of the unit. The activity gives you an opportunity to adjust the lesson based on students' strengths and weaknesses. It also gives students an idea of what topics will be covered in upcoming lessons.

As time permits, have a class discussion about the terms the students just brainstormed. Access students' prior knowledge about the relationships between them. For example, how does a foot compare to a yard?

Questions for Students

How are the terms that you listed related to one another? What guidelines did you use to classify your terms?

> [Student responses may vary. They may classify according to length, width, volume, etc., or according to Metric vs. Customary.]

How and when have you used these types of measurement?

> [Student responses may vary.]

Are there any terms of measurement that were new to you? If so, which ones? What did you learn in this lesson about appropriate uses of these terms?

> [Student responses may vary.]

Assessment Options

1. At this stage of the unit, students should be able to do the following:
   - Understand major terms associated with measurement
   - Know how these terms of measurement relate to one another
   - Know how certain measurements are used in the real world

   This activity aids in planning and pacing the remainder of this unit. Keep the results from this activity to determine how students can add to or adjust the lists that they created. Students will need to refer back to their brainstormed lists in a future lesson.

Teacher Reflection

1. Were the students able to make connections between their own experiences and the words they generated in their brainstorming?

NCTM Standards and Expectations

Measurement 6-8

1. Understand both metric and customary systems of measurement.

This lesson was developed by Katie Carbone.

ILLUMINATIONS
Resources for Teaching Math

Measuring Up

Do You Measure Up?

Students learn the basics of the metric system. They identify which units of measurement are used to measure specific objects, and they learn to convert between units within the same system.

Learning Objectives

Students will:

- identify the appropriate unit of measure for familiar objects and distances
- make conversions between units of measure in metric units

Materials

Index cards
Markers
Metric Conversions Activity Sheet

Instructional Plan

Provide students with index cards and markers. Have them brainstorm different measures used in the metric system and record them on index cards.

In pairs, have the students organize the index cards into categories of weight, length, and volume. Ask the students to rank each unit within the category according to size (smallest to largest).

A sample chart appears below.

Length	Mass	Volume
millimeter	milligram	milliliter
centimeter	gram	liter
meter	kilogram	
kilometer		

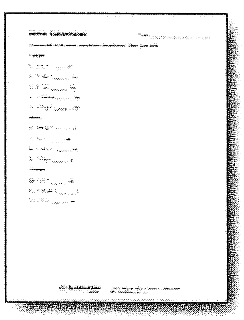
Next, have the students generate a list of items measured with each unit. Place each item on a separate index card and distribute the cards among the class.

A sample list is shown below.

Length	Mass	Volume
millimeter insect lengths	milligram medicines	milliliter medicine dosage
centimeter length of paper	gram food	liter milk
meter length of a hallway	kilogram mass of humans	
kilometer distance between cities		

After the students are familiar with the different sizes of the metric units, show them a conversion table. Refer to the Metric Equivalents lesson plan on the Educator's Reference Desk Web site for an explanation of the conversion method by multiplying and dividing by powers of ten.

Distribute the Metric Conversions activity sheet to the students to work on individually or in pairs.



Metric Conversions Activity Sheet

Questions for Students

What kind of metric units are used to measure fruit at the grocery store?

[Mass of fruit: grams.]

What is the smallest unit of the metric system that you might use to measure length? What is the largest?

[Millimeter; Kilometer]

How is volume measured in the metric system? Where have you seen this unit on everyday products?

[Liter is the base unit; bottles of water]

Where are some common places you have seen the metric system used instead of customary units?

[Student responses may vary]

Assessment Options

1. At this stage of the unit, students should be able to do the following:
   - Identify the appropriate unit of measure for familiar objects and distances
   - Make conversions between units of measure in metric units
2. Tell students to create a series of problems, using the metric system, that they can exchange with each other. Include with each problem an open-ended response question that measures the students' understanding of the metric measure and its application.

Teacher Reflection

- Did the students demonstrate an understanding of metric measures? What additional experiences do they need before moving on to the next lesson?
- Did students apply metric measures correctly? In what areas did they demonstrate confusion about appropriate units for specific tasks?
- What additional experiences do the students need in converting units between customary and metric measures? What additional activities can reinforce this lesson?

NCTM Standards and Expectations

Measurement 6-8

1. Understand both metric and customary systems of measurement.
2. Understand relationships among units and convert from one unit to another within the same system.

This lesson was developed by Katie Carbone.

Metric Conversions

NAME ______________________________

Perform each of the metric conversions shown below. Show your work.

Length

1. 8 km = ________ m

2. 95 mm = ________ cm

3. 6.7 m = ________ cm

4. 1,268 m = ________ km

5. 120 cm = ________ mm

Mass

6. 500 mg = ________ g

7. 9 g = ________ mg

8. 2,500 g = ________ kg

9. 7.8 kg = ________ g

Volume

10. 3.4 L = ________ mL

11. 4,500 mL = ________ L

12. 123 L = ________ mL

ILLUMINATIONS
Resources for Teaching Math

Measuring Up

Off the Scale

Students use real-world examples to solve problems involving scale as they examine maps of their home states and calculate distances between cities.

Learning Objectives

Students will:

- set up and solve problems dealing with scale by writing proportions
- recognize examples of scales being used in real-life situations
- examine a map of their home states to determine distances between cities using a scale

Materials

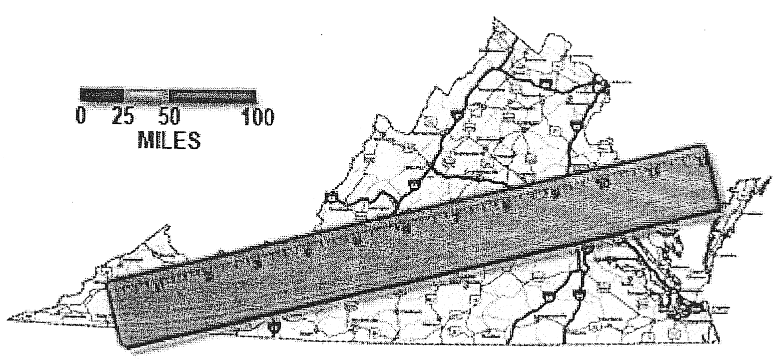
Overhead Transparencies and Pens

Maps of birth states of students
Rulers
Map Activity Sheet

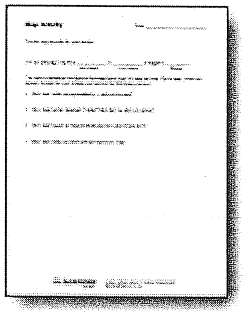
Instructional Plan

To assess students' prior knowledge, have the students brainstorm ideas about where they might use a scale to enlarge or reduce the size of something. List these ideas on the board or on an overhead projector.

To begin the lesson, give the students a copy of their state map and have them locate the legend. Maps of individual states are available at Maps of the United States. Another site is located here. Alternatively, students can find their own state map.



Give pairs of students a ruler and have them figure out distances between given cities. Use the Teacher Resource Sheet, Map activity sheet, as a guideline to creating your own worksheet.



Map Activity Sheet

Students should use the map's scale to complete information about the miles and the inches. Work with the students on setting up proportions to compute unknown distances using their scale factors. Be sure that when the students are setting up the proportions, they do actual distances over each other, and the scale distances over each other (the scale distances or one of the actual distances will be represented by x). Once the proportions are set up, the students can simply solve for x in their proportions.

Questions for Students

What mathematics are involved in enlarging something? Reducing something?

> [Proportions, similarity, scale factor.]

What mathematics do you use to convert inches to miles (on the map) using the scale on the map?

> [Scale, conversion factor].

What steps do you take to convert miles to feet? How about miles to yards?

> [To convert miles to feet, divide miles by 5,280. To convert miles to yards, divide miles by 1,760.]

Assessment Options

1. At this point in the unit, students should be able to do the following:
   - Set up and solve proportions
   - Apply proportions in a variety of instances
   - Apply what they have learned about making conversions and use distances on maps to convert their units to actual distances
2. You can assess these objectives by verbally asking the students to glance at an atlas and use their conversion factors to approximate the distance between certain cities. Once they make an estimate, have students set up their proportions to verify their guesses.

Extensions

1. You can extend the activity by having the students select a comic strip from the newspaper. They should mark off their comic in ¼-inch squares. If they write light enough with their pencil, they can erase the grid after the comic is drawn.

   Then have students mark off an 8 1/2 × 11 inch sheet of paper in one-inch squares. Have students draw each box from their comic onto their paper, enlarging the comic strip. You may consider drawing out a one-inch by one-inch grid on a transparency to show the students exactly how to measure and mark the grid with their rulers.

Teacher Reflection

1. Are the students able to make reasonable estimates using the scale in the map's legend?
2. Are the students able to connect the use of scales to real-world situations?
3. Is further review of the metric and customary conversion factors necessary?

NCTM Standards and Expectations

Measurement 6-8

1. Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.
2. Solve problems involving scale factors, using ratio and proportion.
3. Use common benchmarks to select appropriate methods for estimating measurements.
4. Understand both metric and customary systems of measurement.
5. Understand relationships among units and convert from one unit to another within the same system.

This lesson was developed by Katie Carbone.

Map Activity

NAME ______________________________

Use the map provided by your teacher.

You are planning a trip from _____________ to _____________ on Highway ___________.
(city name) (city name) (Route)

You want to determine the distance between these cities by using the map. On the map, locate the legend showing the scale of miles and answer the following questions.

1. How many miles are represented by 1 inch on the map?

2. How many inches represent 5 miles? How did you get your answer?

3. How many inches are there between the two cities listed above?

4. How many miles are there between these two cities?

Topic 5: Dimensional Analysis

for use after ***Comparing and Scaling*** **Investigation 3**

To convert a measurement from one unit to another, you can use a conversion factor. A **conversion factor** is a rate equal to 1. For example, 12 in. = 1 ft, so you can use the rate $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert feet to inches.

Problem 5.1

A. 1. Use the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert 100 feet to inches.

2. Use a conversion factor to convert 100 inches to feet.

B. 1. What conversion factor can you use to change seconds to minutes?

2. What conversion factor can you use to change minutes to seconds?

C. Which unit belongs in the denominator of the conversion factor, the given measurement or the resulting measurement?

Dimensional analysis is a method of checking the units that result from using conversion factors. You can use dimensional analysis to check whether your methods and answers are reasonable.

Problem 5.2

A. 1. Use conversion factors for hours to minutes and minutes to seconds to write a rate for converting hours to seconds.

2. Write a conversion factor for changing seconds to hours.

B. 1. You want to convert 1,000 seconds to hours. Which method below is correct?

$1{,}000 \text{ s} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 3{,}600{,}000 \text{ h}$

$1{,}000 \text{ s} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.28 \text{ h}$

2. You want to convert 240 miles per second to miles per hour. Which method below is correct?

$240 \frac{\text{mi}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 864{,}000 \text{ mi/h}$

$240 \frac{\text{mi}}{\text{s}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.67 \text{ mi/h}$

Exercises

1. The table shows equivalent measurements.

Measurements

Length in Meters	Length in Feet
1	3.28
2	6.56
3	9.84
4	13.12

   a. Write a conversion factor for changing meters to feet.

   b. Write a conversion factor for changing feet to meters.

   c. How many feet equal 100 meters?

   d. How would you find a conversion factor for changing square meters to square feet?

For Exercises 2–3 below, do parts (a) and (b).

   a. Use a conversion factor to solve the problem.

   b. Use dimensional analysis to check your answer.

2. Change 432 square inches to square feet.

3. Change 2,232 minutes to days.

4. You bike for 45 minutes at a rate of 10 mi/h. You turn around and return by the same route. Your return trip takes 30 minutes. What was your average speed over the entire trip?

5. *Density* is a unit rate. It is the mass of a substance per unit volume. The table gives data for the masses and volumes of four metal samples.

Valuable Metals

Metal	Mass (kilograms)	Volume (cubic centimeters)
Copper	8,930	1
Gold	9,660	0.5
Silver	20,980	2
Titanium	4,500	1

   a. Which metal has the greatest density?

   b. Convert the density of copper from 8,930 kilograms per cubic meter to grams per cubic centimeter. Use dimensional analysis to check that your answer is reasonable.

   c. Which sample below shows the correct first step for converting the density of titanium to grams per cubic centimeter?

   1. $4{,}500 \frac{\text{kg}}{\text{m}^3} \times \frac{1{,}000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ m}}{100 \text{ cm}}$

   2. $4{,}500 \frac{\text{kg}}{\text{m}^3} \times \frac{1 \text{ kg}}{1{,}000 \text{ g}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}}$ ______

   d. Write the density of titanium in grams per cubic centimeter.

Topic 5: Dimensional Analysis

At a Glance

PACING 1 day

Mathematical Goals

- Use conversion factors to convert units
- Use dimensional analysis to check units for reasonableness

Teaching Guide

Students may have trouble deciding whether to multiply or divide when converting between units. In Topic 5, students will learn to use dimensional analysis to check the units of a converted quantity. Students can avoid writing unreasonable answers by writing out all conversion factors and canceling units correctly.

After Problem 5.1, ask:

- *When you convert from a smaller unit to a larger unit, will the numerical result be greater than or less than the original measure?*
- *When you convert from a larger unit to a smaller unit, will the numerical result be greater than or less than the original measure?*
- *Is it easier to think of a length as 5 feet or 60 inches?*

Summarize Problem 5.2A by asking:

- *How do you know that the rate you found for converting hours to seconds is a conversion factor?*
- *How can you use the rate you found for converting hours to seconds to write a conversion factor for changing seconds to hours without writing the conversion factors for hours to minutes and minutes to seconds?*

After Problem 5.2B, ask:

- *How does dimensional analysis help you decide which conversion factors to use?*
- *How can you keep track of which units remain after you multiply by a conversion factor?*

Homework Check

When reviewing Exercise 1, ask:

- *How can you use the table to write conversion factors for changing meters to inches and inches to meters?*

After reviewing Exercises 2–4, ask:

- *Can you write a conversion factor to change square inches to feet? Why or why not?*
- *Can you write a conversion factor to change from minutes to feet? Why or why not?*

Vocabulary

- conversion factor
- dimensional analysis

Assignment Guide for Topic 5

Core 1–5

Answers to Topic 5

Problem 5.1

A. 1. 1,200 inches

2. $8\frac{1}{3}$ ft

B. 1. $\frac{1 \text{ min}}{60 \text{ s}}$

2. $\frac{60 \text{ s}}{1 \text{ min}}$

C. the given measurement

Problem 5.2

A. 1. $\frac{3{,}600 \text{ s}}{1 \text{ h}}$

2. $\frac{1 \text{ h}}{3{,}600 \text{ s}}$

B. 1. the second method, 0.28 h

2. the first method, 864,000 mi/h

Exercises

1. a. $\frac{3.28 \text{ ft}}{1 \text{ m}}$

b. $\frac{1 \text{ m}}{3.28 \text{ ft}}$

c. 328 ft

d. Square the conversion factor for changing meters to feet.

2. a. 3 ft^2

b. Check students' work. Sample: $432 \text{ in.}^2 \times \frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ ft}}{12 \text{ in.}}$

3. a. 1.55 days

b. Check students' work. Sample:

$2{,}232 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ h}}$

4. 12 mi/h

5. a. gold

b. 8.93 g/cm^3

c. 1

d. 4.5 g/cm^3

NCTM ILLUMINATIONS
Resources for Teaching Math

Big Math and Fries

We are lucky to live in an age where there is a lot of nutrition information available for the food we eat. The problem is that much of the data is expressed in percents and some of those percents can be misleading. This lesson is designed to enlighten students about how to calculate percent of calories from fat, carbohydrates, and protein. The calculations are made to determine if a person can follow the Zone Diet with only McDonald's food items.

Learning Objectives

Students will:

- mathematically analyze the food they eat.
- identify the relationship between nutrients and the calories.
- perform calculations, including percents and conversions.

Materials

- McDonald's nutrition facts (or any fast food chain)
- Computer with Internet connection (optional)
- Calculators
- Big Math and Fries Activity Sheet

Instructional Plan

In this lesson, students pick out a full day's worth of meals from a McDonald's menu with an eye towards the Zone Diet, which specifies percentages of fat, carbohydrates, and protein. The gist of the Zone Diet is that whenever you eat, you should strive to consume 40% carbohydrates, 30% protein and 30% fat. As a result, this diet has also been referred to as the 40–30–30 diet.

Many sources indicate that an average person requires about 2,000 calories per day. That number varies based on several factors, but is used as the target for this lesson. To prepare for the lesson, you might research some details on the Internet. For example, you can find out what various athletes consume in a day. You will find that it's much more than 2,000 calories. There was an urban legend floating around for a while that Michael Phelps (Olympic swimming gold medalist) was consuming 12,000 calories per day. That turned out to be untrue, but it might be interesting to start things off with a classroom discussion regarding how many calories various athletes consume. That could lead into a discussion regarding whether students know how many calories they consume and what proportion of nutrients are contained in the foods they eat.

Another lesson opener could be the portrayal of diet in the media. Some students may have seen the movie Supersize Me. Discuss the nutritional concerns about eating fast food. This can naturally progress into a discussion of diets. Suggest the idea of being "in the Zone." Ask whether any students have noticed a relationship between what they eat and how they feel. Do they sometimes feel sleepy? hyper? or just right?

Depending on time, you may wish to have a longer discussion about nutrition. If you choose to do this, you may wish to share the following information on nutrients with students:

- Carbohydrates: our main source of energy
- Fats: one source of energy and important in relation to fat soluble vitamins
- Minerals: inorganic elements that are critical to normal body functions
- Proteins: essential to growth and repair of muscle and other body tissues
- Roughage: the fibrous indigestible portion of our diet essential to health of the digestive system
- Vitamins: important in many chemical processes in the body
- Water: essential to normal body function, both as a vehicle for carrying other nutrients and because 60% of the human body is water

In nutrition, some information focuses on food weight and other information focuses on calories. Make students aware of this before they begin the activity to help them avoid errors based on these units. There is not a one-to-one relationship between food weight and calories. The Zone Diet percentages are all with regard to calories, so if you only have weight information, you need to convert to calories to match the Zone Diet percentages.

Fortunately, the conversions between food weight and calories are simple and students may have already studied this information in physical education or health class. Here are the conversions, which are also provided on the activity sheet:

- Fat: 1 gram = 9 calories
- Carbohydrates: 1 gram = 4 calories
- Protein: 1 gram = 4 calories

This information can be written on the board or put up via a transparency.

Hand out the Big Math and Fries activity sheet and calculators. For this lesson, students should use calculators because of the number of calculations required. You can choose how many decimal places that they should round to. Just remember that some of their calculations will be converted from decimal to percent, so they'll need at least two decimal places for those calculations. Students should also be given McDonald's nutrition facts. You can either hand out paper copies or display McDonald's nutrition information via a computer projector.

PDF Big Math and Fries Activity Sheet

Have students read the McDonald's nutrition pamphlet and try to pick out enough food so that the total number of calories adds up to 2,000 for the day. It is difficult to meet all the caloric and Zone diet requirements at once, so suggest to students to begin with only one or two. Students can attempt to ensure that their percent of calories from fat for the day is less than 30%. If students can do that, they've done well. Then challenge students who succeed to additional goals, such as keeping carbohydrates to 40% of the total calories and keeping protein to 30% of the total calories. It is difficult to achieve all three, but students should be able to keep fat under 30%. More advanced students may be able to get close to the proper percentage for all three nutrients. When more advanced students finish, have them help slower students who are not finished yet.

A nice wrap-up for this lesson would be to have students that came closest to achieving Zone proportions present their findings and explain how they achieved their results. You might also have students could create posters to present their findings and explain why they would recommend the meal combinations that they came up with.

Questions for Students

- Were you able to stay under 30% for total calories from fat? Do you feel that you designed a healthy day of eating?

  [Answers will vary]

- What steps did you take in order to meet the requirements of 2,000 calories total and a 40-30-30 ratio?

  [Answers will vary. What you're looking for are the strategies that students used to try to balance the results. For instance, did successful students focus on one nutrient, get the appropriate percentage and then change one food item to balance the other nutrients? Did they first calculate the total amount of grams needed for each nutrient base on a 2,000 calorie diet and then work backwards? Or did they come up with something new and unique?]

- If you were not able to meet the Zone Diet requirements of 40-30-30, could you tweak a few items to change that? If so, which items would you change and how does that improve your carbohydrates-protein-fat ratio for the day?

  [Students should look at nutrient percentages that are too high and try to figure out which items they could remove or replace in order to get better ratios.]

- Would it be easier to design one Zone friendly meal and, if so, which items would you choose?

  [Yes, it would probably be easier to design just one meal to meet the ratios. This should lead students to think about balancing nutrient ratios when they go to eat a meal or a snack.]

- If you were to design the McDonald's nutrition pamphlet, what would you change from the current design?

  [Answers will vary. One suggestion might be to provide percentage of each nutrient, not just fat.]

Assessment Options

1. Ask students to design a single meal and see how close they can get to the 40-30-30 ratio.
2. Allow students to design a day's meals using any food they choose to meet the Zone Diet. Students should gather their own nutrition information and provide calculations for how they met the diet's restrictions.
3. Remove the caloric restriction from the activity. Just using the Zone Diet restrictions, is the activity easier, harder, or the same?

Extensions

1. Watch the movie, "Supersize Me," with the students and discuss how the movie relates to the lesson. Have students explain whether or not the movie was a fair representation and why. If your students are familiar with sampling and statistical analysis, you may also discuss the experimental model used in the movie.
2. Talk to a health or physical education teacher and see if they have a unit on nutrition where this lesson could be used as a complement.
3. This lesson is well suited for a spreadsheet application or a graphing calculator program. A spreadsheet application might take the grams of each nutrient for each desired item and automatically calculate the percentage of calories. Students could then more easily use trial and error to find the desired ratio of nutrients for a particular meal. This is only recommended in a class familiar with spreadsheets or graphing calculators.

Teacher Reflection

- Were most students able to adjust their food choices to achieve Zone proportions? If not, what problems did they run into? How could these problems be avoided?
- Were any students confused about the difference between calculating the calories due to the nutrient weights and calculating the percent of calories from a particular nutrient? If so, how could this confusion be avoided in the future?
- Were students motivated to achieve Zone proportions or did they just pick various menu items to get the work part over with?
- Did students feel that this lesson was interesting or of use to them?

NCTM Standards and Expectations

Number & Operations 6-8

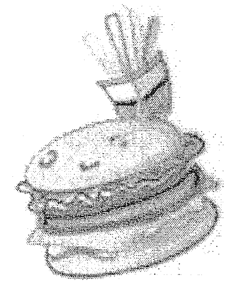
1. Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
2. Work flexibly with fractions, decimals, and percents to solve problems.
3. Understand and use ratios and proportions to represent quantitative relationships.

This lesson was prepared by Michael Weingarden.

Big Math and Fries

NAME________________________

The Zone Diet specifies that a person should eat foods in the ratio of 40 percent carbohydrates, 30 percent protein, and 30 percent fat. Can a person eat just McDonald's food on the Zone Diet?



1. Complete the data table on the next page. Look through the McDonald's menu and create a menu for one full day. Choose all the items you would need to eat to consume about 2,000 calories. Divide the items into meals. The meals could be breakfast, lunch, and dinner, but it is not mandatory. Fill in the Nutrition Facts in your table. Then, calculate the remaining values under Calculated Values using the conversions below:

Fat:	1 gram = 9 calories
Carbohydrates:	1 gram = 4 calories
Protein:	1 gram = 4 calories

2. Were you able to keep the percent of calories from fat under 30% of the total calories?

3. Did you manage to stay within the limits of the Zone Diet? If not, how close did you come to achieving the 40-30-30 ratio?

4. What mathematical strategies did you use in creating your menu? What part of the Zone Diet was most difficult to keep?

	Menu Item	Nutrition Facts				Calculated Values					
		Calories	Fat (g)	Carbo-hydrates (g)	Protein (g)	Fat		Carbohydrates		Protein	
						Cal	% Cal	Cal	% Cal	Cal	% Cal
Meal 1											
Meal 2											
Meal 3											
Total											

ILLUMINATIONS
Resources for Teaching Math

McDonald's USA Nutrition Facts for Popular Menu Items

provide a nutrition analysis of our menu items to help you balance your McDonald's meal with other foods you eat. Our goal is to provide you with the information u need to make sensible decisions about balance, variety and moderation in your diet.

Nutrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE Vitamin A	Vitamin C	Calcium	Iron
Sandwiches																						
Hamburger	3.5 oz (100 g)	250	80	9	13	3.5	16	0.5	25	9	520	22	31	10	2	6	6	12	0	2	10	15
Cheeseburger	4 oz (114 g)	300	110	12	19	6	28	0.5	40	13	750	31	33	11	2	7	6	15	6	2	20	15
Double Cheeseburger	5.8 oz (165 g)	440	210	23	35	11	54	1.5	80	26	1150	48	34	11	2	8	7	25	10	2	25	20
McDouble	5.3 oz (151 g)	390	170	19	29	8	42	1	65	22	920	38	33	11	2	7	7	22	6	2	20	20
Quarter Pounder®+	6 oz (169 g)	410	170	19	29	7	37	1	65	22	730	30	37	12	2	10	8	24	2	4	15	20
Quarter Pounder® with Cheese+	7 oz (198 g)	510	230	26	40	12	61	1.5	90	31	1190	50	40	13	3	11	9	29	10	4	30	25
Double Quarter Pounder® with Cheese++	9.8 oz (279 g)	740	380	42	65	19	95	2.5	155	52	1380	57	40	13	3	11	9	48	10	4	30	35
Big Mac®	7.5 oz (214 g)	540	260	29	45	10	50	1.5	75	25	1040	43	45	15	3	13	9	25	6	2	25	25
Big N' Tasty®	7.2 oz (206 g)	460	220	24	37	8	42	1.5	70	23	720	30	37	12	3	11	8	24	6	8	15	25
Big N' Tasty® with Cheese	7.7 oz (220 g)	510	250	28	43	11	54	1.5	85	28	960	40	38	13	3	12	8	27	10	8	20	25
Angus Bacon & Cheese	10.2 oz (291 g)	790	350	39	60	17	87	2	145	49	2070	86	63	21	4	14	13	45	10	4	25	35
Angus Deluxe	11.1 oz (314 g)	750	350	39	60	16	82	2	135	45	1700	71	61	20	4	16	10	40	15	8	25	35
Angus Mushroom & Swiss	10 oz (283 g)	770	360	40	61	17	85	2	135	46	1170	49	59	20	4	16	8	44	8	0	40	35
Filet-O-Fish®	5 oz (142 g)	380	170	18	28	3.5	18	0	40	14	640	27	38	13	2	7	5	15	2	0	15	10
McChicken ®	5 oz (143 g)	360	150	16	25	3	15	0	35	11	830	34	40	13	2	7	5	14	0	2	10	15
McRib ®†	7.4 oz (209 g)	500	240	26	40	10	48	0	70	23	980	41	44	15	3	10	11	22	2	2	15	20
Premium Grilled Chicken Classic Sandwich	8 oz (226 g)	420	90	10	15	2	10	0	70	23	1190	50	51	17	3	13	11	32	4	10	8	20
Premium Crispy Chicken Classic Sandwich	8.1 oz (230 g)	530	180	20	31	3.5	17	0	50	17	1150	48	59	20	3	13	12	28	4	8	8	20
Premium Grilled Chicken Club Sandwich	8.8 oz (250 g)	530	160	17	27	6	29	0	95	31	1410	59	52	17	4	14	12	39	8	10	20	20
Premium Crispy Chicken Club Sandwich	9 oz (254 g)	630	250	28	43	7	36	0	75	25	1360	57	60	20	4	14	13	35	8	8	20	20

Premium Grilled Chicken Ranch BLT Sandwich	8.3 oz (236 g)	470	110	12	19	3	15	0	80	27	1440	60	54	18	3	14	12	36	4	10	10	20
Premium Crispy Chicken Ranch BLT Sandwich	8.5 oz (240 g)	580	200	23	35	4.5	22	0	65	21	1400	58	62	21	3	14	13	31	4	8	8	20
Southern Style Crispy Chicken Sandwich	8.7 oz (161 g)	400	150	17	26	3	14	0	45	16	1030	43	39	13	1	5	6	24	2	2	10	10
Ranch Snack Wrap® (Crispy)	4.1 oz (117 g)	340	150	17	26	4.5	23	0	30	10	810	34	33	11	1	4	2	14	2	0	10	10
Ranch Snack Wrap® (Grilled)	4.3 oz (122 g)	270	90	10	16	4	19	0	45	15	830	35	26	9	1	4	2	18	2	2	10	10
Honey Mustard Snack Wrap® (Crispy)	4.2 oz (118 g)	330	140	16	24	4.5	22	0	30	10	780	33	34	11	1	4	4	14	2	0	10	10
Honey Mustard Snack Wrap® (Grilled)	4.4 oz (124 g)	260	80	9	14	3.5	18	0	45	15	800	33	27	9	1	4	4	18	2	2	10	10
Chipotle BBQ Snack Wrap® (Crispy)	4.2 oz (120 g)	330	140	15	24	4.5	22	0	30	9	810	34	35	12	1	5	4	14	4	0	10	10
Chipotle BBQ Snack Wrap® (Grilled)	4.4 oz (125 g)	260	80	9	13	3.5	18	0	45	14	830	34	28	9	1	5	5	18	4	2	10	10
Mac Snack Wrap†	4.4 oz (126 g)	330	170	19	30	7	34	1	45	15	690	29	26	9	1	5	3	15	2	0	8	15

Nutrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE			
																			Vitamin A	Vitamin C	Calcium	Iron
French Fries																						
Small French Fries	2.5 oz (71 g)	230	100	11	18	1.5	8	0	0	0	160	7	29	10	3	12	0	3	0	8	2	4
Medium French Fries	4.1 oz (117 g)	380	170	19	29	2.5	13	0	0	0	270	11	48	16	5	20	0	4	0	15	2	6
Large French Fries	5.4 oz (154 g)	500	220	25	38	3.5	17	0	0	0	350	15	63	21	6	26	0	6	0	20	2	8
Ketchup Packet	1 pkg (10 g)	15	0	0	0	0	0	0	0	0	110	5	3	1	0	0	2	0	2	2	0	0
Salt Packet	1 pkg (0.7 g)	0	0	0	0	0	0	0	0	0	270	11	0	0	0	0	0	0	0	0	0	0
Nutrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE			
																			Vitamin A	Vitamin C	Calcium	Iron
Chicken McNuggets®/Chicken Selects® Premium Breast Strips/Sauces																						
Chicken McNuggets® (4 piece)	2.3 oz (64 g)	190	100	12	18	2	10	0	30	9	400	17	11	4	0	0	0	10	0	2	0	4
Chicken McNuggets® (6 piece)	3.4 oz (95 g)	280	160	17	27	3	15	0	40	14	600	25	16	5	0	0	0	14	0	2	2	4

Chicken McNuggets® (10 piece)	5.6 oz (159 g)	460	260	29	44	5	25	0	70	23	1000	42	27	9	0	0	0	24	0	2	2	8
rbeque Sauce	1 pkg (28 g)	50	0	0	0	0	0	0	0	0	260	11	12	4	0	0	10	0	2	0	0	0
Honey	1 pkg (14 g)	50	0	0	0	0	0	0	0	0	0	0	12	4	0	0	11	0	0	0	0	0
Hot Mustard Sauce	1 pkg (28 g)	60	20	2.5	4	0	0	0	5	1	250	10	9	3	2	8	6	1	0	0	0	2
Sweet 'N Sour Sauce	1 pkg (28 g)	50	0	0	0	0	0	0	0	0	150	6	12	4	0	0	10	0	2	0	0	0
Chicken Selects® Premium Breast Strips (3 pc)	4.6 oz (131 g)	400	210	24	37	3.5	17	0	50	17	1010	42	23	8	0	0	0	23	0	0	2	4
Chicken Selects® Premium Breast Strips (5 pc)	7.7 oz (219 g)	660	360	40	61	6	28	0	85	29	1680	70	39	13	0	0	0	38	0	0	4	8
Spicy Buffalo Sauce	1.3 oz (35 g)	60	50	6	9	1	5	0	0	0	800	33	1	0	1	6	0	0	4	2	0	0
Creamy Ranch Sauce	1.3 oz (35 g)	170	160	18	28	3	14	0	10	3	270	11	2	1	0	0	1	0	0	0	0	0
Tangy Honey Mustard Sauce	1.3 oz (35 g)	60	20	2	3	0	0	0	5	2	140	6	10	3	0	0	8	0	0	0	0	0
Southwestern Chipotle Barbeque Sauce	1.3 oz (5 g)	60	0	0	0	0	0	0	0	0	210	9	15	5	1	2	11	0	4	0	2	2

utrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE			
																			Vitamin A	Vitamin C	Calcium	Iron
Salads																						
Premium Southwest Salad with Grilled Chicken	12.3 oz (350 g)	320	80	9	14	3	14	0	70	24	960	40	30	10	6	25	11	30	130	50	15	15
Premium Southwest Salad with Crispy Chicken	12.5 oz (353 g)	430	180	20	30	4	20	0	55	18	920	38	38	13	6	25	12	26	130	50	15	15
Premium Southwest Salad (without chicken)	8.1 oz (231 g)	140	40	4.5	7	2	9	0	10	3	150	6	20	7	6	24	6	6	130	45	15	10
Premium Bacon Ranch Salad with Grilled Chicken	11.3 oz (321 g)	260	90	9	15	4	21	0	90	30	1010	42	12	4	3	13	5	33	130	50	15	10
Premium Bacon Ranch Salad with Crispy Chicken	11.4 oz (324 g)	370	180	20	31	6	28	0	75	24	970	40	20	7	3	13	6	29	130	50	15	10
Premium Bacon Ranch Salad (without chicken)	7.8 oz (223 g)	140	70	7	11	3.5	18	0	25	9	300	12	10	3	3	13	4	9	130	50	15	8
mium Caesar lad with Grilled Chicken	11 oz (311 g)	220	60	6	10	3	15	0	75	25	890	37	12	4	3	13	5	30	130	50	20	10
Premium Caesar Salad with Crispy Chicken	11.1 oz (314 g)	330	150	17	26	4.5	22	0	60	19	840	35	20	7	3	13	6	26	130	50	20	10

Premium Caesar Salad (without chicken)	7.5 oz (213 g)	90	35	4	6	2.5	12	0	10	4	180	7	9	3	3	13	4	7	130	50	20	8
Side Salad	3.1 oz (87 g)	20	0	0	0	0	0	0	0	0	10	0	4	1	1	6	2	1	45	25	2	4
Butter Garlic Croutons	0.5 oz (14 g)	60	15	1.5	3	0	0	0	0	0	140	6	10	3	1	2	0	2	0	0	2	4
Snack Size Fruit & Walnut Salad	1 pkg (163 g)	210	70	8	13	1.5	7	0	5	2	60	2	31	10	2	9	25	4	0	170	8	2

Nutrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE Vitamin A	Vitamin C	Calcium	Iron
Salad Dressings																						
Newman's Own® Creamy Southwest Dressing	1.5 fl oz (44 ml)	100	50	6	9	1	5	0	20	7	340	14	11	4	0	0	3	1	0	0	2	2
Newman's Own® Creamy Caesar Dressing	2 fl oz (59 ml)	190	170	18	28	3.5	17	0	20	7	500	21	4	1	0	0	2	2	0	0	6	0
Newman's Own® Low Fat Balsamic Vinaigrette	1.5 fl oz (44 ml)	40	25	3	4	0	0	0	0	0	730	30	4	1	0	0	3	0	0	4	0	0
Newman's Own® Low Fat Family Recipe Italian Dressing	1.5 fl oz (44 ml)	60	20	2.5	4	0	0	0	0	0	730	30	8	3	0	0	1	1	0	0	0	0
Newman's Own® Ranch Dressing	2 fl oz (59 ml)	170	130	15	23	2.5	12	0	20	6	530	22	9	3	0	0	4	1	0	0	4	0

Nutrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE Vitamin A	Vitamin C	Calcium	Iron
Breakfast																						
Egg McMuffin®	7.1 oz (137 g)	300	110	12	19	5	24	0	260	87	820	34	30	10	2	8	3	18	10	0	30	20
Sausage McMuffin®	6.2 oz (111 g)	370	200	22	34	8	42	0	45	15	850	35	29	10	2	8	2	14	6	2	25	15
Sausage McMuffin® with Egg	8 oz (162 g)	450	250	27	42	10	51	0	285	95	920	38	30	10	2	8	2	21	10	2	30	20
English Muffin	4.3 oz (56 g)	160	30	3	5	0.5	3	0	0	0	280	12	27	9	2	7	2	5	2	0	15	10
Bacon, Egg & Cheese Biscuit (Regular Size Biscuit)	4.9 oz (140 g)	420	210	23	35	12	59	0	235	79	1160	48	37	12	2	7	3	15	10	0	15	15
Bacon, Egg & Cheese Biscuit (Large Size Biscuit)	5.4 oz (154 g)	480	240	27	42	12	62	0	235	79	1270	53	43	14	3	12	4	15	15	0	15	20
Sausage Biscuit with Egg (Regular Size Biscuit)	5.7 oz (163 g)	510	290	33	50	14	71	0	250	83	1170	49	36	12	2	6	2	18	6	0	10	20

Sausage Biscuit with Egg (Large Size Biscuit)	6.2 oz (177 g)	570	330	37	57	15	74	0	250	83	1280	53	42	14	3	11	3	18	10	0	10	20
Sausage Biscuit (Regular Size Biscuit)	4.1 oz (117 g)	430	240	27	42	12	62	0	30	10	1080	45	34	11	2	6	2	11	0	0	6	15
Sausage Biscuit (Large Size Biscuit)	4.6 oz (131 g)	480	280	31	48	13	65	0	30	10	1190	50	39	13	3	11	3	11	4	0	8	15
Southern Style Chicken Biscuit (Regular Size Biscuit)	5 oz (143 g)	410	180	20	31	8	41	0	30	10	1180	49	41	14	2	6	3	17	0	2	6	15
Southern Style Chicken Biscuit (Large Size Biscuit)	5.5 oz (157 g)	470	220	24	37	9	45	0	30	10	1290	54	46	15	3	11	4	17	4	2	8	15
Biscuit (Regular Size)	2.7 oz (76 g)	260	110	12	18	7	35	0	0	0	740	31	33	11	2	6	2	5	0	0	6	10
Biscuit (Large Size)	3.2 oz (90 g)	320	140	16	25	8	38	0	0	0	850	36	39	13	3	11	3	5	4	0	6	15
Bacon, Egg & Cheese McGriddles®	6.3 oz (164 g)	420	160	18	28	8	38	0	240	80	1110	46	48	16	2	8	15	15	10	0	20	15
Sausage, Egg & Cheese McGriddles®	7.6 oz (201 g)	560	290	32	49	12	61	0	265	88	1360	56	48	16	2	8	15	20	10	0	20	15
Sausage McGriddles®	5 oz (141 g)	420	200	22	34	8	40	0	35	11	1030	43	44	15	2	8	15	11	0	0	8	10
Big Breakfast® (Regular Size Biscuit)	9.5 oz (269 g)	740	430	48	73	17	87	0	555	185	1560	65	51	17	3	12	3	28	15	2	15	25
Big Breakfast® (Large Size Biscuit)	10 oz (283 g)	800	470	52	80	18	90	0	555	185	1680	70	56	19	4	17	3	28	15	2	15	30
Big Breakfast with Hotcakes (Regular Size Biscuit)	14.8 oz (420 g)	1090	510	56	87	19	96	0	575	192	2150	90	111	37	6	23	17	36	15	2	25	40
Big Breakfast with Hotcakes (Large Size Biscuit)	15.3 oz (434 g)	1150	540	60	93	20	100	0	575	192	2260	94	116	39	7	28	17	36	15	2	30	40
Sausage Burrito	3.9 oz (111 g)	300	140	16	25	7	33	0.5	130	43	830	35	26	9	1	4	2	12	10	2	15	15
McSkillet™ Burrito with Sausage	8.4 oz (238 g)	610	320	36	56	14	69	0.5	410	137	1390	58	44	15	3	11	4	27	20	10	20	25
McSkillet™ Burrito with Steak	9.8 oz (250 g)	570	270	30	46	12	59	1	430	143	1470	61	44	15	3	11	4	32	20	10	20	30
Hotcakes (w/o Syrup & Margarine)	5.3 oz (151 g)	350	80	9	13	2	9	0	20	7	590	24	60	20	3	10	14	8	0	0	15	15
Hotcakes and Sausage (w/o Syrup & Margarine)	6.8 oz (192 g)	520	210	24	37	7	36	0	50	17	930	39	61	20	3	10	14	15	0	0	15	15
Hotcake Syrup	1 pkg (60 g)	180	0	0	0	0	0	0	0	0	20	1	45	15	0	0	32	0	0	0	0	0
Whipped Margarine (1 pat)	6 g	40	40	4.5	7	1.5	8	0	0	0	55	2	0	0	0	0	0	0	4	0	0	0

Sausage Patty	1.4 oz (41 g)	170	140	15	23	5	27	0	30	10	340	14	1	0	0	0	0	7	0	0	2	2
Scrambled Eggs (2)	3.3 oz (96 g)	170	100	11	17	4	19	0	520	174	180	7	1	0	0	0	0	15	15	0	6	10
sh Brown	2 oz (56 g)	150	80	9	14	1.5	6	0	0	0	310	13	15	5	2	6	0	1	0	2	0	2
Grape Jam	0.5 oz (14 g)	35	0	0	0	0	0	0	0	0	0	0	9	3	0	0	9	0	0	2	0	0
Strawberry Preserves	0.5 oz (14 g)	35	0	0	0	0	0	0	0	0	0	0	9	3	0	0	9	0	0	4	0	0

Nutrition Facts	**Serving Size**	**Calories**	**Calories from Fat**	**Total Fat (g)**	**% Daily Value****	**Saturated Fat (g)**	**% Daily Value****	***Trans* Fat (g)**	**Cholesterol (mg)**	**% Daily Value****	**Sodium (mg)**	**% Daily Value****	**Carbohydrates (g)**	**% Daily Value****	**Dietary Fiber (g)**	**% Daily Value****	**Sugars (g)**	**Protein (g)**	**% DAILY VALUE** Vitamin A	Vitamin C	Calcium	Iron
Desserts/Shakes																						
Fruit 'n Yogurt Parfait (7 oz)»	5.3 oz (149 g)	160	20	2	3	1	5	0	5	2	85	4	31	10	1	3	21	4	0	15	15	4
Fruit 'n Yogurt Parfait (without granola) (7 oz)»	5 oz (142 g)	130	15	2	3	1	5	0	5	2	55	2	25	8	0	0	19	4	0	15	10	2
Low Fat Caramel Dip	0.8 oz (21 g)	70	5	0.5	1	0	0	0	5	1	35	2	15	5	0	0	9	0	0	0	2	0
Vanilla Reduced Fat Ice Cream Cone	3.2 oz (90 g)	150	35	3.5	6	2	11	0	15	5	60	2	24	8	0	0	18	4	6	0	10	2
die Cone	1 oz (29 g)	45	10	1	2	0.5	4	0	5	2	20	1	8	3	0	0	6	1	2	0	4	0
Strawberry Sundae	6.3 oz (178 g)	280	60	6	10	4	20	0	25	8	95	4	49	16	1	6	45	6	10	4	20	0
Hot Caramel Sundae	6.4 oz (182 g)	340	70	8	12	5	25	0	30	10	160	7	60	20	1	6	44	7	10	0	25	0
Hot Fudge Sundae	6.3 oz (179 g)	330	90	10	15	7	35	0	25	8	180	8	54	18	2	8	48	8	10	0	25	6
Peanuts (for Sundaes)	0.3 oz (7 g)	45	30	3.5	5	0.5	3	0	0	0	0	0	2	1	1	2	0	2	0	0	0	0
McFlurry® with M&M'S® Candies (12 fl oz cup)	12.5 oz (353 g)	710	230	25	39	16	78	1	60	19	220	9	105	35	4	16	97	15	20	0	50	8
McFlurry® with OREO® Cookies (12 fl oz cup)	11.6 oz (329 g)	580	170	19	30	10	50	1	50	17	320	14	89	30	3	14	73	13	20	0	45	8
Baked Hot Apple Pie	2.7 oz (77 g)	250	110	13	19	7	35	0	0	0	170	7	32	11	4	15	13	2	4	25	2	6
Cinnamon Melts	4 oz (114 g)	460	170	19	30	9	43	0	15	5	370	15	66	22	3	11	32	6	4	0	6	15
McDonaldland® Cookies	2 oz (57 g)	260	70	8	13	2.5	12	0	0	0	300	12	43	14	1	3	13	4	0	0	0	10
Chocolate Chip Cookie	1 cookie (33 g)	160	70	8	12	3.5	19	0	10	3	90	4	21	7	1	3	15	2	2	0	2	8
meal Raisin Cookie	1 cookie (33 g)	150	50	6	9	2.5	13	0	10	3	135	6	22	7	1	3	13	2	2	0	2	6
Sugar Cookie	1 cookie (33 g)	160	60	7	11	3	15	0	5	2	120	5	21	7	0	0	11	2	2	0	0	4

Nutrition Facts	Serving Size	Calories	Calories from Fat	Total Fat (g)	% Daily Value**	Saturated Fat (g)	% Daily Value**	*Trans* Fat (g)	Cholesterol (mg)	% Daily Value**	Sodium (mg)	% Daily Value**	Carbohydrates (g)	% Daily Value**	Dietary Fiber (g)	% Daily Value**	Sugars (g)	Protein (g)	% DAILY VALUE			
																			Vitamin A	Vitamin C	Calcium	Iron
Beverages																						
1% Low Fat Milk Jug	1 carton (236 ml)	100	20	2.5	4	1.5	8	0	10	3	125	5	12	4	0	0	12	8	10	4	30	0
1% Low Fat Chocolate Milk Jug	1 carton (236 ml)	170	25	3	4	1.5	9	0	5	2	150	6	26	9	1	3	25	9	10	6	30	0
Minute Maid® Apple Juice Box	6.8 fl oz (200 ml)	100	0	0	0	0	0	0	0	0	15	1	23	8	0	0	22	0	0	100	10	0
Dasani® Water	16.9 fl oz	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Minute Maid® Orange Juice (Small)§	12 fl oz cup	150	0	0	0	0	0	0	0	0	0	0	30	10	0	0	30	2	0	140	2	0
Minute Maid® Orange Juice (Medium)§	16 fl oz cup	190	0	0	0	0	0	0	0	0	0	0	39	13	0	0	39	3	0	180	4	0
Minute Maid® Orange Juice (Large)	22 fl oz cup	280	0	0	0	0	0	0	0	0	5	0	58	19	0	0	58	4	0	260	4	0
Coca-Cola® Classic (Child)§	12 fl oz cup	110	0	0	0	0	0	0	0	0	5	0	29	10	0	0	29	0	0	0	0	0
Coca-Cola® Classic (Small)§	16 fl oz cup	150	0	0	0	0	0	0	0	0	10	0	40	13	0	0	40	0	0	0	0	0
Coca-Cola® Classic (Medium)§	21 fl oz cup	210	0	0	0	0	0	0	0	0	15	1	58	19	0	0	58	0	0	0	0	0
Coca-Cola® Classic (Large)§	32 fl oz cup	310	0	0	0	0	0	0	0	0	20	1	86	29	0	0	86	0	0	0	0	0
Diet Coke® (Child)§	12 fl oz cup	0	0	0	0	0	0	0	0	0	15	1	0	0	0	0	0	0	0	0	0	0
Diet Coke® (Small)§	16 fl oz cup	0	0	0	0	0	0	0	0	0	20	1	0	0	0	0	0	0	0	0	0	0
Diet Coke® (Medium)§	21 fl oz cup	0	0	0	0	0	0	0	0	0	30	1	0	0	0	0	0	0	0	0	0	0
Diet Coke® (Large)§	32 fl oz cup	0	0	0	0	0	0	0	0	0	45	2	0	0	0	0	0	0	0	0	0	0
Sprite® (Child)§	12 fl oz cup	110	0	0	0	0	0	0	0	0	30	1	28	9	0	0	28	0	0	0	0	0
Sprite® (Small)§	16 fl oz cup	150	0	0	0	0	0	0	0	0	40	2	39	13	0	0	39	0	0	0	0	0
Sprite® (Medium)§	21 fl oz cup	210	0	0	0	0	0	0	0	0	55	2	56	19	0	0	56	0	0	0	0	0
Sprite® (Large)§	32 fl oz cup	310	0	0	0	0	0	0	0	0	80	3	83	28	0	0	83	0	0	0	0	0
Hi-C® Orange Lavaburst (Child)§	12 fl oz cup	120	0	0	0	0	0	0	0	0	0	0	32	11	0	0	32	0	0	110	0	0
Hi-C® Orange Lavaburst (Small)§	16 fl oz cup	160	0	0	0	0	0	0	0	0	5	0	44	15	0	0	44	0	0	150	0	0

Additional Practice Pages

Questions for Students

When would you need to make conversions between units?

[If the students have traveled abroad, ask them about the use of metric measures in other countries. For example, many countries have notebooks based on metric measures.]

When would you use a combination of inches, feet, or yards to measure?

[Student responses may vary.]

What are some things to consider when deciding which unit of measure to choose?

[Are you measuring length, weight, volume, etc? What is the relative "size" of the item being measured?]

Assessment Options

At this stage of the unit, students should be able to do the following:

- Use their conversion factors to set up an equation to convert from one unit of measurement to another
- Recognize the relevance of converting units in everyday life
- Choose the most appropriate unit of measurement in a given situation

Extension

Distribute the *You Think Gas is Expensive?* activity sheet. Allow students to calculate the unit rates on each item while making conversions from ounces to gallons. The items on the activity sheets are things that should be familiar to the students, so the process gives the students an opportunity to use what they have learned and apply this knowledge to everyday products.

Teacher Reflection

- What additional experiences do students who are having difficulties setting up their chart or table need to make conversions?
- Do the students have a good grasp of the units themselves? Do they understand how big the units are relative to one another?
- Do students need more practice measuring objects to understand the different customary units of measure?

NCTM Standards and Expectations

Measurement 6-8:

- Understand both metric and customary systems of measurement.
- Understand relationships among units and convert from one unit to another within the same system.

You Think Gas Is Expensive?

NAME ______________________

Just look at a few of these items you probably use. Using the cost given, compute the cost per gallon. By comparison, is a gallon of gas really that expensive?

ITEM AND COMMON PRICE	PRICE PER GALLON
Regular gasoline, $2.25 for one gallon	$2.25
Diet Snapple® iced tea, $1.29 for 16 ounces	
Lipton® iced tea, $1.19 for 16 ounces	
Gatorade® thirst quencher, $1.59 for 20 ounces	
Ocean Spray® juice, $1.25 for 16 ounces	
Milk, $3.19 per gallon	
Brake fluid, $3.15 for 12 ounces	
Vick's NyQuil® cough medicine, $8.35 for 6 ounces	
Pepto-Bismol® stomach medicine, $3.85 for 5 ounces	
BIC Wite-Out® correction fluid, $1.39 for 7 ounces	
Scope® mouthwash, $0.99 for 1.5 ounces	
Evian® bottled water, $1.49 for 9 ounces	

1. Which item was most expensive per gallon? Did that surprise you?

2. What items could you add to the list? Which is the most expensive item when you determine the price per gallon?

Name ______________________ Class ______________ Date ______________

Enrichment 5-2

Unit Rates and Proportional Reasoning

Critical Thinking

The table on the right shows exchange rates for several foreign currencies.

Country	Currency	Rate Per Dollar
Mexico	Peso	7.52
Britain	Pound	0.64
Germany	Mark	1.53
France	Franc	5.18

1. For each country, give the exchange rate as a unit rate in foreign currency per dollar. Then find an equivalent rate to show how much foreign currency you would receive for the given amount of U. S. dollars. The first one is done for you.

	Unit rate	Foreign currency
a. Mexican pesos, 10 dollars	______________	______________
b. British pounds, 5 dollars	______________	______________
c. German marks, 7 dollars	______________	______________
d. French francs, 3 dollars	______________	______________

2. Suppose your company wants to sell a software package in Mexico and in Germany. It sells for $250 in the United States. What will you charge in each country?

   a. Write an equivalent ratio to show how many pesos are in $250.

   ____

   b. Write an equivalent ratio to show how many marks are in $250.

   ____

3. Suppose your marketing manager travels from Great Britain to Germany to meet you.

   a. Write a rate comparing German marks to British pounds. Then give the unit rate per pound. Round your answer to the nearest hundredth.

   ____

   b. Write an equivalent ratio to show the cost in German marks of a software package that costs 48 British pounds.

   ____

Name ________________________________ Period __________

Measures of Capacity and Mass -- Metrics

Walking The Metric Path!

Move decimal to the left! ← BASE UNIT → Move decimal to the right!

kL	hL	dkL	L	dL	cL	mL
kilo-	hecto-	deca-	liter gram	deci-	centi-	milli-
kg	hg	dkg	g	dg	cg	mg

Complete.

1. 1 mL = __________ L
2. 125 mg = __________ g
3. 10 mL = __________ L
4. 125 mg = __________ kg
5. 100 mL = __________ L
6. 525 g = __________ mg
7. 5 L = __________ mL
8. 525 g = __________ kg
9. 50 L = __________ mL
10. 7 kg = __________ g
11. 67 mL = __________ L
12. 4 L = __________ mL
13. 83 g = __________ mg
14. 260 mg = __________ kg
15. 0.9 mg = __________ g
16. 39 kg = __________ g
17. 1,633 g = __________ kg
18. 8,750 mL = __________ kL
19. 32 mg = __________ g
20. 9.4 g = __________ mg

Write the better measurement.

21. capacity of a bathtub	20 mL	2,000 mL	20 L
22. capacity of a glass	200 mL	2,000 mL	200 L
23. capacity of an eyedropper	2 mL	2,000 mL	2 L
24. mass of a grain of salt	5 mg	5 g	5 kg
25. mass of a pair of shoes	2 mg	2 g	2 kg

Solve.

26. A jug of apple juice is 0.0025 L. What is the capacity of the jug in milliliters?

____

27. A large egg has a mass of about 60 grams. What is the mass of a dozen eggs? Answer using kilograms.

____

Name ______________________________ Period __________

Equal Distances -- Metrics

Walking The Metric Path!

← Move decimal to the left!

Move decimal to the right! →

			BASE UNIT			
kilo-	**hecto-**	**deca-**	**meter**	**deci-**	**centi-**	**milli-**
km	hm	dam	m	dm	cm	mm
kilometer	hectometer	dekameter	meter	decimeter	centimeter	millimeter

Complete.

1. 8 cm = ________ mm
2. 16 mm = ________ m
3. 25 dm = ________ cm
4. 8 cm = ________ dm
5. 42 cm = ________ km
6. 9 dam = ________ cm
7. 7 m = ________ dam
8. 9 dam = ________ km
9. 16 mm = ________ dam
10. 42 cm = ________ mm
11. 1.600 mm = ________ km
12. 23 hm = ________ m
13. 5 m = ________ hm
14. 30 cm = ________ m
15. 5 m = ________ km
16. 25 dm = ________ m
17. 4 dm = ________ cm
18. 6 hm = ________ cm
19. 9 dam = ________ m
20. 3 km = ________ m
21. 182.5 m = ________ cm
22. 25 dm = ________ dam
23. 6 hm = ________ km
24. 42 cm = ________ km
25. 16 mm = ________ cm
26. 7 dam = ________ km
27. 88 cm = ________ mm
28. 6.7 cm = ________ m
29. 16 mm = ________ dm
30. 5 m = ________ dam

31. A desk that is 2,250 mm long = ________ m.

32. A fence that is 270 cm long = ________ m.

33. A dog that is 675 mm long = ________ cm.

34. A stretch of highway that is 3,200 m long = ________ km.

Accentuate the Negative

6th graders in '08 – '09 did up through division of integers so as 7th graders in '09 – '10 they only need a review. HOWEVER, in '10 – '11, 7th graders will need more formal teaching if not received as 6th graders in '09 – '10.

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations /Additional Targets
Topic 1 Prime Factorization	1	Online lesson		
Investigation 1 Extending the Number Line: 1.1 (1 day), 1.2 (1 day), 1.3 1 day, Mathematical reflections p. 17 (1 day)	4			**7.1.A Compare and order rational numbers using the number line, lists, and the symbols, <, >, or =**
Topic 2 Comparing Numbers and Absolute Value	1	Online lesson		
Walk the Number Line Activity	1	binder		
Investigation 2 Adding Integers: 2.1 (1 day), 2.2 (2 day) with Skill: Adding Integers p. 70 as homework assignment	3		Must do	*7.1.B Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.*
Check-Up 1 P. 84, 85 teacher edition	1			
Investigation 3 Subtracting Integers: 3.1 (2 days), 3.2 (2 days), 3.3 (1 day)	5			**7.1.C Fluently and accurately add, subtract, multiply, and divide rational numbers.**
CMP2 Skill: Subtracting Integers Inv 2 p.71, Skill: Adding & Subtracting Rational Numbers Inv 2 p. 72	1	Binder/CMP2 Disc 2(7)		
Mathematical Reflection p. 52, **Check-Up 2 p. 86 (**teacher)	1			**7.1.D Define and determine the absolute value of a number.**
Investigation 4 Multiplying & Dividing Integers: 4.1 (1 day), 4.2 (1 day), 4.3 (1 day), 4.4 (1 day)	4			
CMP2 Skill: Multiplying Integers, p. 76 and Dividing Integers, p. 77	2	CMP2 Disc(7)		*7.2.A Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.*
CMP2 Skill: Multiplying and Dividing Rational Numbers p. 78				
Investigation 5: Coordinate Grids: 5.1 (1 day)	1	CMP2 Disc(7)		
CMP2 Skill: Coordinate Graphing p.73 **plus** *hand out blank coordinate grid and have students plot some points with rationals (include pos/ neg fractions and decimals)*	1	Binder/CMP2 Disc (7)		**7.5.A Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.**
Accentuate the Negative Assessment	1			
Additional Practice pages at end of section				
Review & Reflect Assessment, Student Self-Assessment	1			**7.5.B Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Total Instructional Days for Integer Review:	**28**			

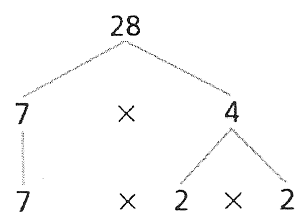
Contents in Accentuate the Negative

Topic 1: Prime Factorization

Use to begin ***Accentuate The Negative***

Any composite number can be written as a product using its factors. The **prime factorization** of a number is the product of prime factors equal to that number.

You can use factor trees to find the prime factorization of a number. For example, to make a factor tree for 28, begin by choosing two numbers whose product is 28. Continue dividing each number into two factors until each of the branches ends in a prime number. Below is a sample factor tree for 28.



The prime factorization of 28 is $2 \times 2 \times 7$, or $2^2 \times 7$.

The small raised number in $2^2 \times 7$ is an exponent. An **exponent** tells how many times a factor is multiplied repeatedly. For example, the expression $3^2 \times 5^3$ means $3 \times 3 \times 5 \times 5 \times 5$.

Problem 1.1

A. **1.** List all of the factors of 90. Which factors are prime?

2. Which factor pairs could you use to start a factor tree for 90?

3. Make two different factor trees for 90. What do you notice about the prime factorization of each tree?

4. How can you write the prime factorization of 90 using exponents?

B. **1.** Are each of the prime factors of a number included in the prime factorization at least once?

2. Does it matter which two factors you choose for the first line of a factor tree?

Exercises

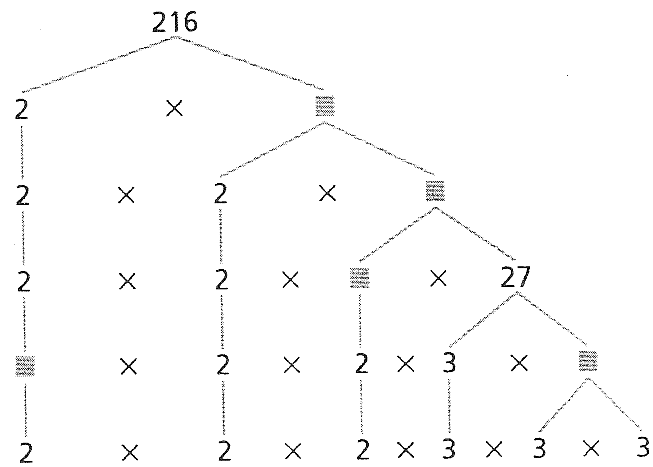
For Exercises 1–9, find the prime factorization of each number.

1. 20
2. 36
3. 50
4. 85
5. 100
6. 189
7. 525
8. 639
9. 1,000

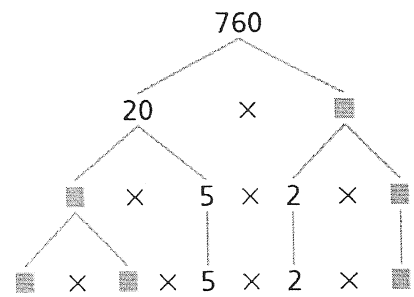
10. Use exponents to write the prime factorizations you found in Exercises 1–9.

For Exercises 11–12, copy and complete the factor trees.

11.



12.



For Exercises 13–16, consider the prime factorization of each of the whole numbers from 2 through 50.

13. Which of the whole numbers from 2 to 25 are prime?

14. Which of the whole numbers from 20 to 30 are the product of exactly three different prime factors?

15. Which of the whole numbers from 30 to 40 have 5 as a prime factor?

16. Which of the whole numbers from 40 to 50 have a prime factorization with only odd prime factors?

17. **a.** Make three different factor trees for 360.

    b. How many different pairs of numbers could you use to start a factor tree for 360?

Topic 1: Prime Factorization

At a Glance

PACING 1 day

Mathematical Goals

- Use a factor tree to find a prime factorization
- Write a prime factorization using exponents

Teaching Guide

Vocabulary

- prime factorization
- exponent

Before beginning Topic 1, review the concepts of prime numbers and factors with students. You can also review the concept of exponents with students. Explain that an expression in the form a^b is called a power; a is the base, and b is the exponent.

Before Problem 1.1, ask:

- *What is the definition of a prime number?*
- *Is 1 considered a prime number?*
- *How can you find the factors of a number?*
- *How can you use exponents to rewrite the expression* $5 \times 5 \times 5$*? the expression* $2 \times 2 \times 5 \times 5 \times 5$*?*

During Problem 1.1, you may want to review divisibility rules to help the students find factors. Ask:

- *How do you know if a number is divisible by 2? by 3? by 5?*
- *How do you know if an number is divisible by 6? by 10?*

After Problem 1.1B, explain to students that the prime factorization of a number is unique. Students may be unsure about the order in which they should write the numbers in a prime factorization. Explain that the prime factors are usually written with primes listed in increasing order. Point out to students that the order of the prime factors does not change the prime factorization, because multiplication is commutative. Remind students to include repeated prime factors in a prime factorization, or to use exponent notation.

Homework Check

When reviewing Exercise 17, ask:

- *Is there a factor pair that cannot be used to start a factor tree?*
- *Does the factor pair you choose to start a factor tree matter? Will you always get the same result?*

Assignment Guide for Topic 1

Core 1–17

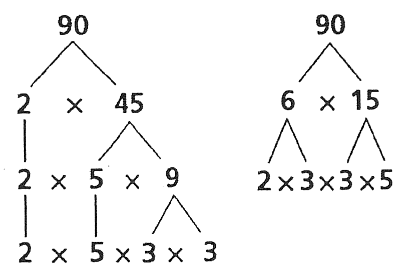
Answers to Topic 1

Problem 1.1

A. 1. Factors of 90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
   Prime factors of 90: 2, 3, 5

2. 2 and 45, 3 and 30, 5 and 18, 6 and 15, 9 and 10

3. Answers may vary. Sample:



4. $2 \times 3^2 \times 5$

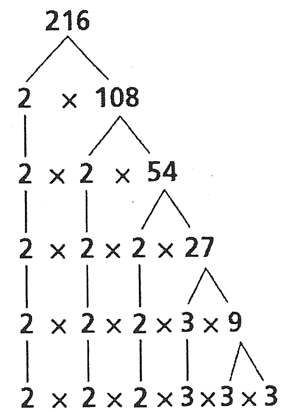
B. 1. yes

2. no

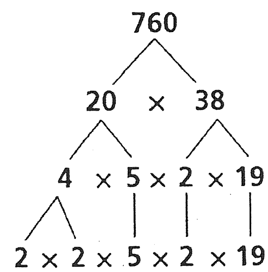
Exercises

1. $2 \times 2 \times 5$
2. $2 \times 2 \times 3 \times 3$
3. $2 \times 5 \times 5$
4. 5×17
5. $2 \times 2 \times 5 \times 5$
6. $3 \times 3 \times 3 \times 7$
7. $3 \times 5 \times 5 \times 7$
8. $3 \times 3 \times 71$
9. $2 \times 2 \times 2 \times 5 \times 5 \times 5$
10. $2^2 \times 5$, $2^2 \times 3^2$, 2×5^2, 5×17, $2^2 \times 5^2$, $3^3 \times 7$, $3 \times 5^2 \times 7$, $3^2 \times 71$, $2^3 \times 5^3$

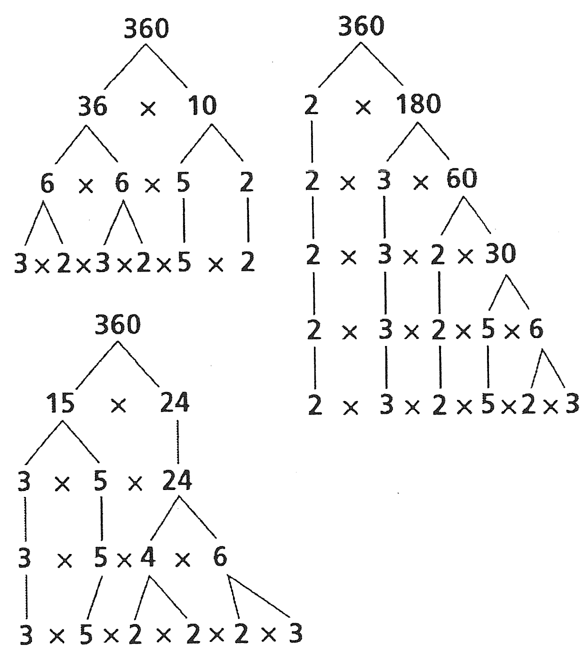
11.



12.



13. 2, 3, 5, 7, 11, 13, 17, 19, 23
14. 21, 22, 26
15. 30, 35, 40
16. 41, 43, 45, 47, 49
17. a. Answers may vary. Sample:



b. 11

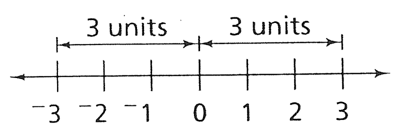
Topic 2: Comparing Numbers and Absolute Value

for use before ***Thinking With Mathematical Models*** **Investigation 1**

Rational numbers are numbers that can be expressed as one integer divided by another non-zero integer. Examples of rational numbers are $\frac{3}{4}$, $-\frac{7}{8}$, $\frac{3}{-1}$, 0.75, and $\sqrt{9}$.

Irrational numbers are numbers that cannot be expressed as one integer divided by another integer. Examples of irrational numbers are $\sqrt{2}$, $-\sqrt{3}$, π and 0.474774777....

The **absolute value** of a number a, represented as $|a|$, is the distance between the number a and zero. Because distance is a measurement, the absolute value of a number is always positive.



Opposites, like –3 and 3, have the same absolute value, 3, because they are each three units from zero.

Problem 2.1

Alexis, Brandon, Jacob, and Madison are playing a game. Each player gets a card with a number on it. The reverse side of the card contains a hidden letter. The four players line up from least to greatest. If they are correct, the hidden letters spell a word.

A. **1.** For Round 1, Alexis has 0, Brandon has –2, Jacob has 2, and Madison has –1. How should the students line up?

2. When they reveal their letters, they spell the word NICE. Assign each letter to the proper student.

3. Which students have numbers that have the same absolute value?

B. **1.** For Round 2, Alexis has $\frac{2}{3}$, Brandon has $-\frac{3}{5}$, Jacob has $-\frac{1}{2}$, and Madison has $\frac{1}{10}$. How should the students line up?

2. When they reveal their letters, they spell AMTH. What mistake do you think they made?

C. For Round 3, Alexis has $\sqrt{7}$, Brandon has $\sqrt{9}$, Jacob has 4, and Madison has 2. Alexis is not quite sure where to stand. Madison tells her that $\sqrt{4}$ is 2 and $\sqrt{9}$ is 3. Where should Alexis stand?

D. The students decide to make another set of cards, $-0.3\overline{3}$, $-\pi$, $-\sqrt{2}$ and $-\frac{1}{3}$.

1. List the numbers from least to greatest
2. Could OOPS be the hidden word?

Exercises

Use the following list of numbers for Exercise 1–5.

$-\frac{1}{2}$	$-2\frac{1}{2}$	π	-0.5	$\sqrt{4}$
4	$\frac{1}{4}$	$\sqrt{12}$	-2	0.25
0	$\sqrt{3}$	$-\sqrt{4}$	$\frac{1}{2}$	2
$-0.3\overline{3}$	-4	$-\frac{3}{5}$	3	2.5

1. List all of the rational numbers.
2. List all of the irrational numbers.
3. Give an approximate location for each number on a number line.
4. Which numbers have the same value?
5. Which numbers have the same absolute value?
6. **a.** Order the numbers $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$, from least to greatest.

   b. As the denominator of a fraction increases, does the resulting positive fraction get larger or smaller?

   c. Does your rule apply for $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}$? Explain.

Topic 2: Comparing Numbers and Absolute Value

At a Glance

PACING 1 day

Mathematical Goals

- Compare rational and irrational numbers by using the symbols $\leqslant$, $\geqslant$, $<$, $>$, and $=$.
- Apply the concept of absolute value.

Guided Instruction

Remind students that the prefix *ir* means "not," so *irrational* means "not rational."

Tell students that every rational number corresponds to a point on the number line. Draw a number line marked in units between –5 and 5. Remind students of the concept of absolute value.

- *Locate -3 on the number line.* (From zero, go 3 units to the left.)
- *Locate $-\frac{7}{8}$ on the number line.* (Between zero and -1, closer to -1; it is $\frac{7}{8}$ units to the left of zero.)
- *What are the absolute values of –3 and $-\frac{7}{8}$?* (3 and $\frac{7}{8}$, respectively)
- *Compare -3 and $-\frac{7}{8}$ using* $<$. ($-3 < -\frac{7}{8}$)
- *Locate $\sqrt{9}$ on the number line.* (Because $\sqrt{9} = 3$, go 3 units to the right.)
- *Is π a rational number?* (No) *Why not?* (It cannot be expressed as a ratio of two integers.)
- *Can π be located on a number line?* (Not exactly, only an approximation of π can be located as a point on the line.)
- *Give your best estimate as to where to locate π on the number line.* (π is approximately 3.14, so just to the right of 3.)
- *Compare $\sqrt{9}$ and π using* $>$. ($\pi > \sqrt{9}$)

When solving Problem 2.1, have four students act out the problem. Provide cards with additional numbers and letters to give students additional practice ordering rational and irrational numbers.

You will find additional work on comparing rational numbers in the grade 6 unit *Bits and Pieces I.*

Vocabulary

- rational numbers
- irrational numbers
- absolute value

ACE Assignment Guide for Topic 2

Core 1–6

Answers to Topic 2

Problem 2.1

A. 1. Brandon, Madison, Alexis, Jacob

2. Brandon, N; Madison, I; Alexis, C; Jacob, E

3. Brandon and Jacob

B. 1. Brandon, Jacob, Madison, Alexis

2. Answers may vary. Sample: Brandon and Jacob got confused with the negative fractions. Since $\frac{3}{5} > \frac{1}{2}$, $-\frac{3}{5} < -\frac{1}{2}$ since it is further from zero.

C. Between 2 and 3, but closer to 3.

D. 1. $-\pi$, $-\sqrt{2}$, then $-\frac{1}{3}$ and -0.33 are equal.

2. No. The word would end in a double letter.

Exercises

1. $-\frac{1}{2}, -2\frac{1}{2}, -0.5, \sqrt{4}, 4, \frac{1}{4}, -2, 0.25, 0, -\sqrt{4}, \frac{1}{2}, 2, -0.3\overline{3}, -4, -\frac{3}{5}, 3, 2.5$

2. $\pi, \sqrt{12}, \sqrt{3}$

3. See Figure 1.

4. $-\sqrt{4} = -2$; $-\frac{1}{2} = -0.5$; $\frac{1}{4} = 0.25$; $2 = \sqrt{4}$

5. $|-\sqrt{4}| = |-2| = |2| = |\sqrt{4}|$;

$|-\frac{1}{2}| = |-0.5| = |\frac{1}{2}|$;

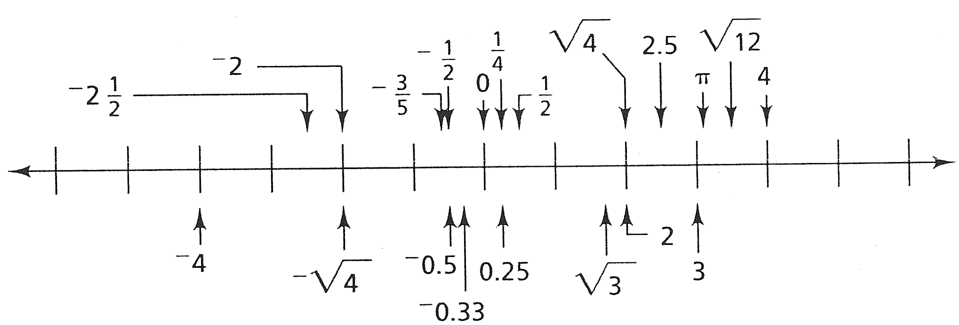
$|\frac{1}{4}| = |0.25|$; $|-4| = |4|$; $|-2\frac{1}{2}| = |2.5|$

6. a. $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

b. The resulting fraction gets smaller.

c. Answers may vary. Sample: No. For negative numbers, an increasing denominator results in larger fractions.

Figure 1



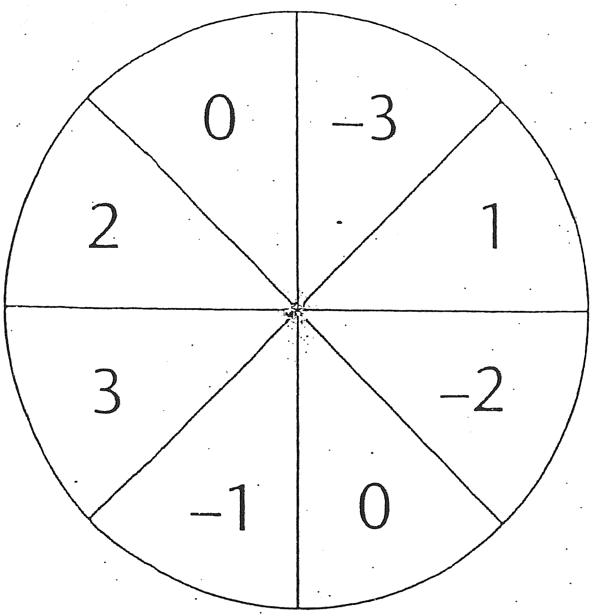
Walk the Line Directions

Divide the class into groups of 4. You will need to assign roles that will rotate after each completion of an equation.
The roles are; 1) the walker 2) the spinner 3) the checker(makes sure the walker is walking correctly) and 4) the monitor (makes sure everyone in the group is writing down the equation).

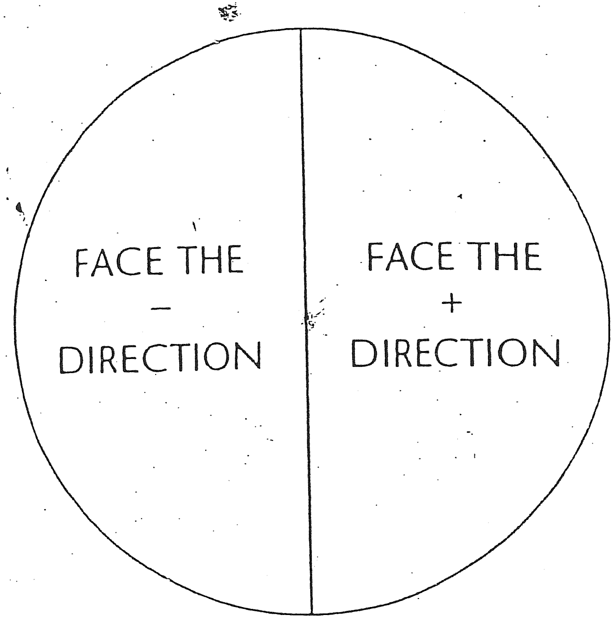
You will need to make 6-8 copies of the masters (preferable on tag or construction paper and then laminate for future use) for a class set. You will need to demonstrate how to do the activity first before the class works in their group. All students need paper and pencil.

- Have each group put their number line on the floor
- Have students use a paper clip and their pencil for the spinner
- Students first spin the starting position for the walker (this number is the 1st number of the equation)
- walker stands on the number
- Students then spin the direction (this tells the walker what direction to face on the number line.)
- Student then spin the move (the walker walks this many spaces on the line and should end on the answer to the equation) **TEACHER NOTE:** the direction spin is the operational sign for the equation.
- If a – number is spun the student walks backwards, if a + number is spun the student walks forward
- For example The number −3 is spun for the start, the direction is a face the – direction and then move −2 the walker starts at −3 faces the negative direction and then walks backwards 2 spaces and ends up at −1
- All the students in the group write down $-3 - (-2) = -1$ Monitor checks for accuracy and that all students are participating
- Rotate positions; spinner is now the walker, checker is the spinner, monitor is the checker, and the walker is the monitor
- You'll need to save the last 10 minutes to summarize the group work. Making sure you discuss when you're facing the negative direction and you spin a negative number why is the end result larger than where you started from.

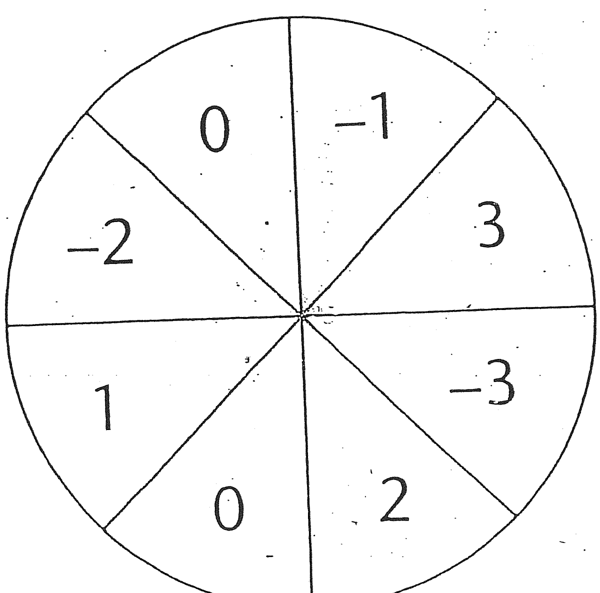
START



DIRECTION



MOVE



9 -

-5

-4

-3

-2

-1

Name ______________________ Date ____________ Class ____________

Skill: Subtracting Integers

Investigation 2

Accentuate the Negative

Find each difference.

1. $9 - 26$
2. $-4 - 15$
3. $21 - (-7)$
4. $27 - (-16)$
5. $-16 - (-43)$
6. $47 - 19$
7. $-156 - 98$
8. $-192 - 47$
9. $0 - (-51)$
10. $-63 - 89$
11. $-12 - (-21)$
12. $92 - (-16)$
13. $72 - 15$
14. $-86 - (-19)$
15. $17 - (-46)$
16. $-78 - (-53)$
17. $-19 - (-12)$
18. $-16 - (-21)$
19. $27 - 19$
20. $-14 - 27$

Name ______________________ Date __________ Class __________

Skill: Adding and Subtracting Rational Numbers

Investigation 2

Accentuate the Negative

Find each sum or difference as a mixed number or fraction in simplest form.

1. $\frac{3}{4} + \frac{7}{8}$

2. $-1\frac{1}{6} + 2\frac{2}{3}$

3. $4\frac{1}{2} - 7\frac{7}{8}$

4. $-3\frac{5}{6} - \left(4\frac{1}{12}\right)$

5. $\frac{5}{18} + \frac{7}{12}$

6. $-4\frac{7}{20} + 3\frac{9}{10}$

7. $5\frac{8}{21} - \left(-3\frac{1}{7}\right)$

8. $1\frac{19}{24} + 2\frac{23}{20}$

9. $3\frac{16}{25} - 4\frac{7}{20}$

Write each answer as a fraction or mixed number in simplest form.

10. $14.6 + \left(-3\frac{1}{5}\right)$

11. $-7\frac{3}{4} + 4.125$

12. $5.75 + \left(-2\frac{1}{8}\right)$

Name ______________________ Date __________ Class __________

Skill: Multiplying Integers

Investigation 3

Accentuate the Negative

Multiply.

1. 7×8
2. -5×7
3. $4 \times (-8)$
4. $-8 \times (-2)$
5. $11 \times (-6)$
6. -7×6
7. $-8 \times (-8)$
8. 10×4
9. 21×13
10. -15×12
11. $-25 \times (-14)$
12. $10 \times (-25)$

For Exercises 13–18, find the missing number.

13. $3 \times \square = -6$
14. $4 \times \square = -4$
15. $\square \times (24) = -8$
16. $-3 \times \square = 9$
17. $-9 \times (-2) = \square$
18. $\square \times (-2) = -18$

19. Your teacher purchases 24 pastries for a class celebration, at $2 each. What integer expresses the amount he paid?

20. Temperatures have been falling steadily at 5°F each day. What integer expresses the change in temperature in degrees 7 days from today?

21. A submarine starts at the surface of the Pacific Ocean and descends 60 feet every hour. What integer expresses the submarine's depth in feet after 6 hours?

22. A skydiver falls at approximately 10 meters per second. Write a number sentence to express how many meters he will fall in 40 seconds.

Name ______________________ Date __________ Class __________

Skill: Dividing Integers

Investigation 3

Accentuate the Negative

Divide.

1. $14 \div 7$
2. $21 \div (-3)$
3. $-15 \div 5$
4. $-27 \div (-9)$
5. $45 \div (-9)$
6. $-42 \div 6$
7. $-105 \div (-15)$
8. $63 \div (-9)$
9. $108 \div 6$
10. $-204 \div 17$
11. $240 \div (-15)$
12. $-252 \div (-12)$

Find each product or quotient.

13. $\frac{-36}{9}$
14. $\frac{-52}{-4}$
15. $(-5) \cdot (-20)$
16. $\frac{-63}{-9}$
17. $(-15) \cdot (2)$
18. $\frac{22}{-2}$
19. $(13) \cdot (-6)$
20. $\frac{-100}{-5}$
21. $(-60) \cdot (-3)$

For Exercises 22 and 23, represent each pattern of change with an integer.

22. spends $300 in 5 days
23. runs 800 feet in 4 minutes

24. Juan's baseball card collection was worth $800. Over the last 5 years, the collection decreased $300 in value. What integer represents the average decrease in value each year?

25. Florence purchased stock for $20 per share. After 6 days, the stock is worth $32 per share. What integer represents the average increase in stock value each day?

Name ______________________ Date __________ Class __________

Skill: Multiplying and Dividing Rational Numbers

Investigation 3

Accentuate the Negative

Use the algorithms you developed to find each value.

1. $-\frac{1}{6} \cdot 2\frac{3}{4}$

2. $\frac{3}{16} \div \left(-\frac{1}{8}\right)$

3. $-\frac{31}{56} \cdot (-8)$

4. $-5\frac{7}{12} \div 12$

5. $-8 \div \frac{1}{4}$

6. $-3\frac{1}{6} \div \left(-2\frac{1}{12}\right)$

7. $8\frac{3}{4} \cdot 3\frac{7}{8}$

8. $-\frac{11}{12} \div \frac{5}{6}$

9. $4\frac{9}{28} \cdot (-7)$

10. $-1\frac{1}{15} \div 15$

11. $-3 \div \frac{3}{4}$

12. $-2\frac{7}{8} \div 3\frac{3}{4}$

13. $-\frac{23}{24} \cdot (-8)$

14. $\frac{7}{8} \cdot \left(-\frac{2}{7}\right)$

15. $-7 \div \frac{1}{9}$

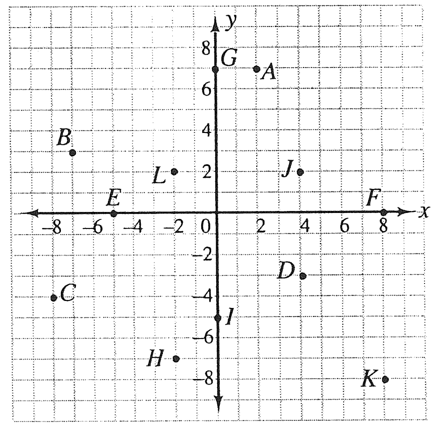
Name ______________________ Date __________ Class __________

Skill: Coordinate Graphing

Investigation 2

Accentuate the Negative

Name the point with the given coordinates.



1. (−2, 2) **2.** (8, 0)

3. (4, −3) **4.** (−7, 3)

5. (0, −5) **6.** (−8, −4)

Write the coordinates of each point.

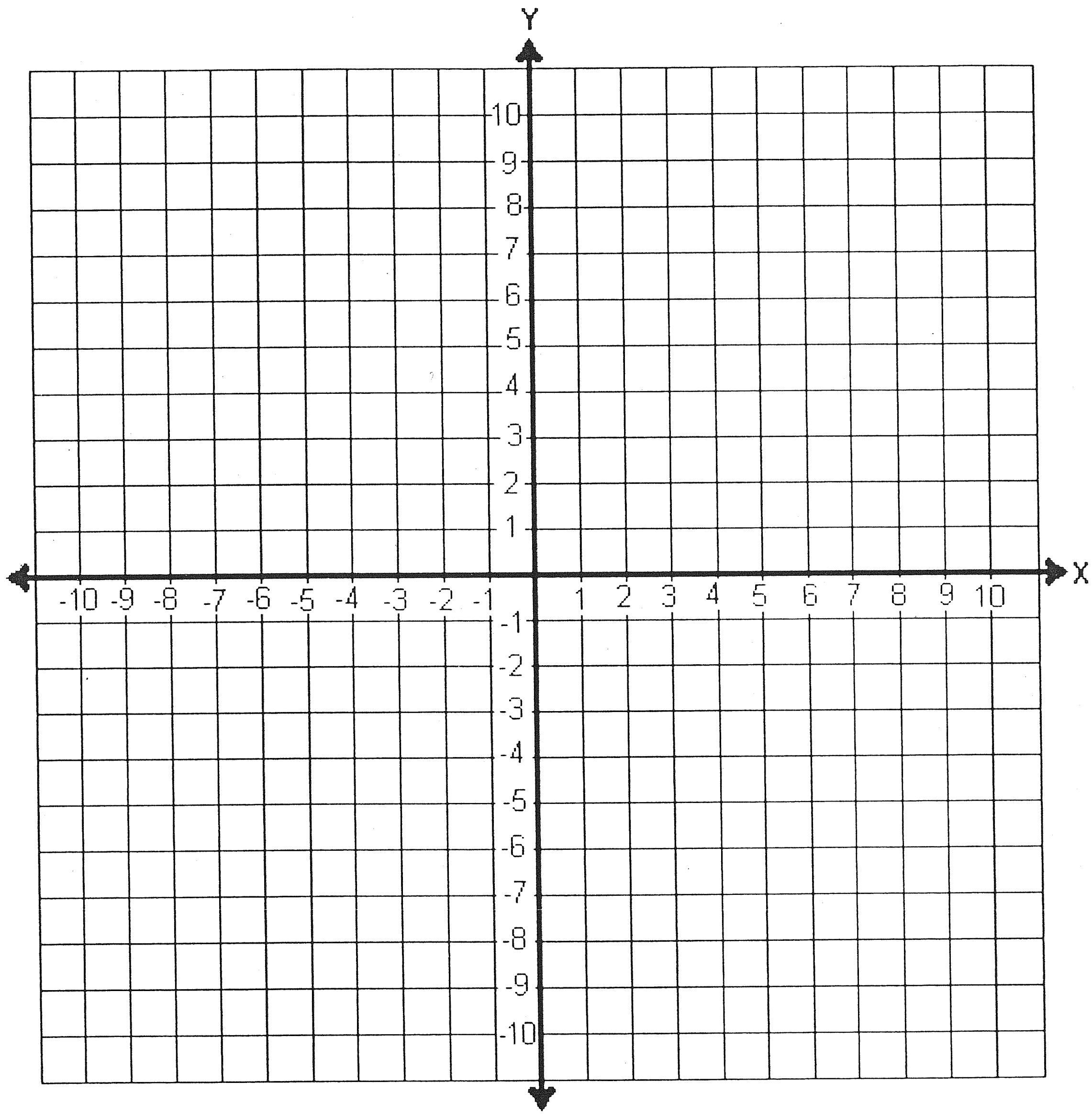
7. *E* **8.** *A*

9. *H* **10.** *K*

11. *G* **12.** *J*

Identify the quadrant in which each point lies.

13. (−4, 3) **14.** (7, 21) **15.** (5, −8) **16.** (−2, −7)


Y
X
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7 8 9 10

Name ______________________ Date __________ Class __________

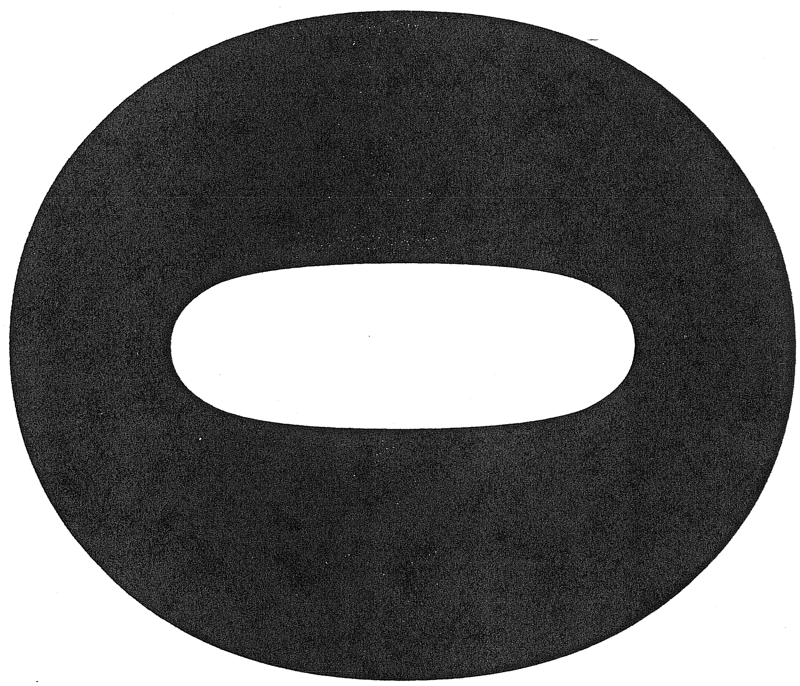
Skill: Adding Integers

Investigation 2

Accentuate the Negative

Simplify each expression.

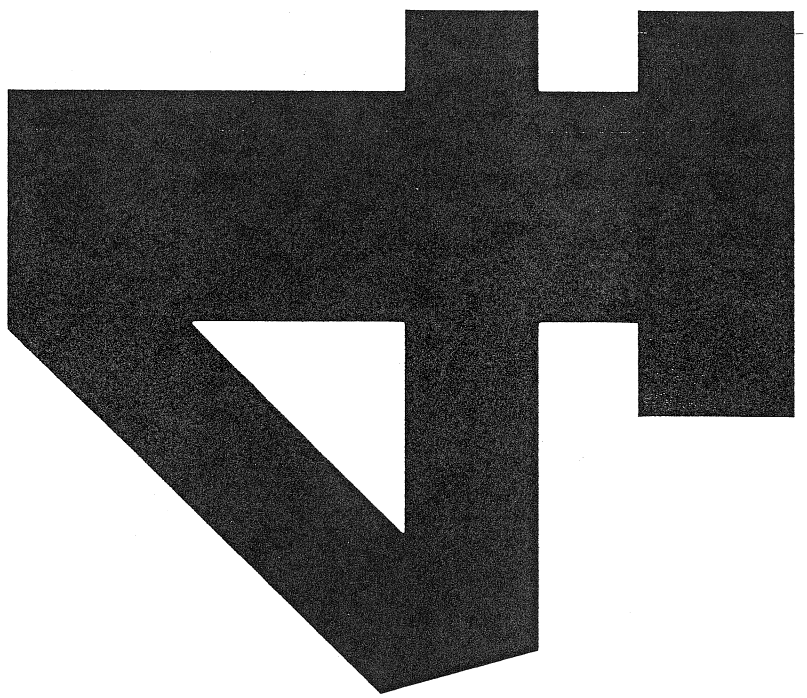
1. $-2 + (-3)$
2. $8 - 7 + 4$
3. $8 + (-5)$
4. $15 + (-3)$
5. $-16 + 8$
6. $7 + (-10)$
7. $-9 + (-5)$
8. $-12 + 14$
9. $8 + 7$
10. $9 + (-4)$
11. $-6 + (-8)$
12. $8 + (-14)$
13. $9 + (-17)$
14. $-15 + (-11)$
15. $-23 + 18$
16. $-19 + 16$
17. $27 + 34$
18. $-8 + (-17)$
19. $19 + (-8)$
20. $23 + (-31)$



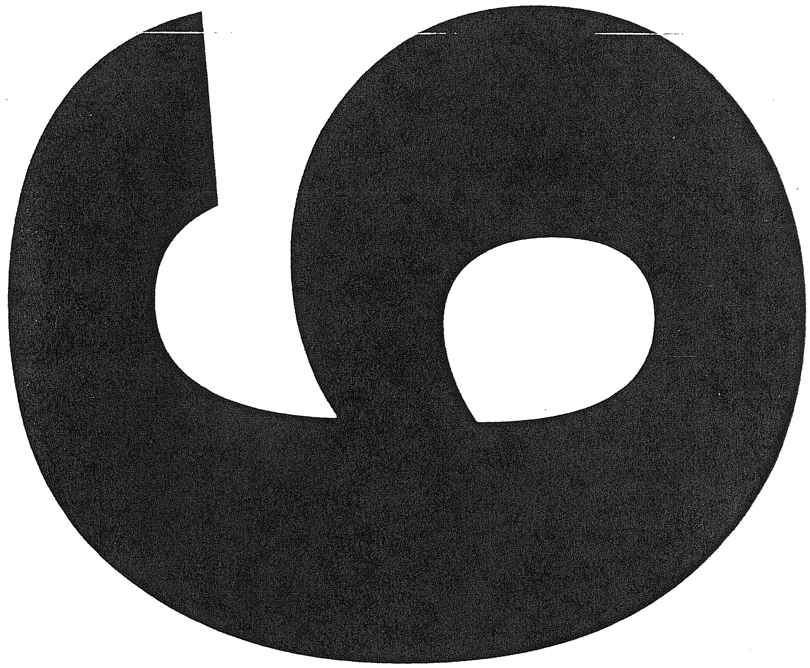
1

2

3



5



POSITIVE DIRECTION

POSITIVE DIRECTION

POSITIVE DIRECTION

NEGATIVE DIRECTION

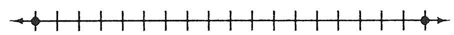
NEGATIVE DIRECTION

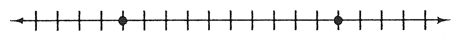
NEGATIVE DIRECTION

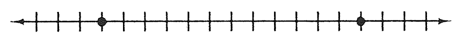
Investigation 1 Additional Practice

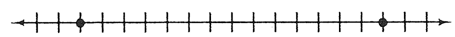
1. a. $^{+}2$ b. $^{+}1$ c. 0 d. $^{-}1$
   e. $^{-}2$ f. $^{-}1$ g. 0 h. $^{+}1$
2. a. 0 b. 0 c. 0 d. 0
   e. All values are 0; there are equal numbers of black and red chips
3. Answers will vary.
4. The value did not change; Sarah added a value equal to 0.
5. The value decreased by 3; there are 3 more red chips than black chips
6. 200 points; answered correctly
7. 150 points; answered incorrectly
8. 150 points; answered incorrectly
9. 250 points; answered incorrectly
10. 250 points; answered incorrectly
11. Possible answers: 9 and $^{-}5$, 4 and $^{-}10$, 0 and $^{-}14$, $^{-}1$ and $^{-}15$
12. Possible answers: 4 and $^{-}2$, 3 and $^{-}3$, 2 and $^{-}4$, 0 and $^{-}6$
13. $^{-}35$ and $^{-}15$ or $^{-}35$ and $^{-}55$
14. $^{-}9$ and 9 **15.** 17 and 8
16. $^{-}2$°F **17.** 13°F **18.** 0°F
19. $^{-}26$°F **20.** $^{-}45$°F **21.** $^{-}25$ m
22. 80 m **23.** −180 m **24.** 110 m
25. The three changes are −15, 55, and −75. The total change is −35. So the submarine ends 35 meters lower than its initial position.
26. The original board had 1 black chip and 3 red chips. The addition sentence would be $^{-}2 + 7 = 5$.
27. The original board had 8 black chips and 7 red chips. The addition sentence would be $1 + {}^{-}5 = {}^{-}4$.
28. The original board had 3 black chips and 1 red chip. The addition sentence would be $2 + 2 + {}^{-}2 = 2$.
29. The original board had 2 black chips and 0 red chips. The addition sentence would be $2 + 5 + {}^{-}8 = {}^{-}1$.
30. The original board had 0 black chips and 3 red chips. The addition sentence would be $^{-}3 + 6 + {}^{-}8 = {}^{-}5$.

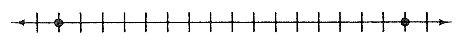
Skill: Integers

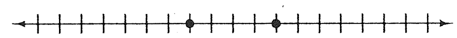
1. −8 **2.** 2 **3.** −4
4. 9 **5.** 7 **6.** −2
7. < **8.** > **9.** <
10. < **11.** > **12.** >
13. 

14. 

15. 

16. 

17. 

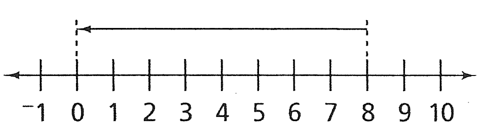
18. 

19. −25, −15, −5, 0, 5, 15, 25
20. −8, −4, −2, 1, 3, 6, 7
21. −18, −10, −8, −6, 3, 9, 27
22. −9, −7, −4, −3, −1, 4, 7
23. No; the highest temperature in Idaho was 118°F.
24. California
25. Yes; Minnesota has a recorded low temperature of −59°.
26. Idaho
27. $7 + (-4) = 3$
28. $-140 + 112 = -28$
29. $18 + (-4) + 12 = 26$
30. $72 + (-12) = 60$

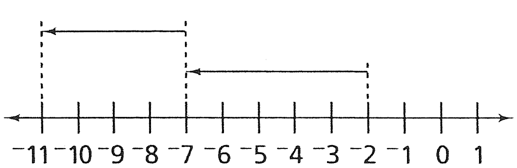
Investigation 2 Additional Practice

1. a. 90 + 60 + 80 + 40 + 50 + 80 = \$400
   b. $^{-}50 + {}^{-}40 + {}^{-}60 + {}^{-}90 + {}^{-}10 + {}^{-}20 = {}^{-}$\$270
   c. \$400 + $^{-}$\$270 = \$130; The store made money.
   d. By comparing the heights of the bars for credit and debit for each month, it is easy to tell which months had profits and which had losses. December was the only month with a loss. September, October, November, January, and February all showed a profit.

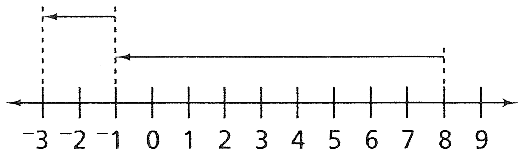
2. Possible answer: Represent $^{-}8$ as 13 red chips and 5 black chips. Remove 5 black chips. The difference is $^{-}13$.
3. Possible answer: Represent 3 as 6 red chips and 9 black chips. Remove 9 black chips. The difference is $^{-}6$.
4. Possible answer: Represent $^{-}6$ as 12 red chips and 6 black chips. Remove 12 red chips. The difference is 6.
5. $^{-}7 + 4 = {}^{-}3$; $^{-}7 - {}^{-}4 = {}^{-}3$
6. $4 + {}^{-}9 = {}^{-}5$; $4 - 9 = {}^{-}5$
7. **a.** 0 **b.** 0 **c.** 0
8. 2 **9.** $^{-}8$ **10.** $^{-}6$ **11.** 4
12. 4 **13.** $^{-}5$ **14.** $^{-}13$ **15.** 2.2
16. $^{-}\frac{1}{3}$ **17.** 7 **18.** $^{-}9$ **19.** $^{-}3$
20. $^{-}4$ **21.** $^{-}7.8$ **22.** 19.3
23. **a.** Always true; Subtracting a positive number from a negative number is equivalent to adding a negative number to a negative number. A negative number plus a negative number is always negative.
b. The statement is sometimes true. Here are some examples:
$5 - 3 = {}^{+}2, 5 - 5 = 0, 5 - 8 = {}^{-}3$
24. **a.** $^{-}5 + {}^{+}2 = {}^{-}3$ other answers are
$^{-}3 - {}^{-}5 = {}^{+}2$ similar
$^{-}3 - {}^{+}2 = {}^{-}5$
25. Pairs of facts like $^{+}5 - {}^{-}2 = {}^{+}7$ and $^{+}7 = {}^{+}5 - {}^{-}2$ are really the same. An equation can be read "either direction."
26. 0



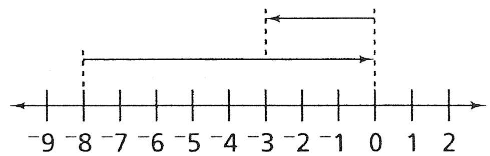
27. $^{-}11$



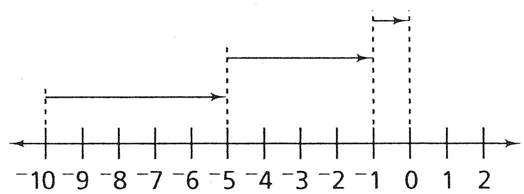
28. $^{-}3$



29. -3



30. 0



31. $^{-}5 + 8 = 3$ **32.** $9 + {}^{-}2 = 7$
33. $9 + {}^{-}11 = {}^{-}2$ **34.** $^{-}6 + {}^{-}3 = {}^{-}9$
35. **a.** $^{-}7 + 4$ is 7 red chips and 4 black chips. $7 + {}^{-}10$ is 7 black chips and 10 red chips.
b. When simplified, both boards have 3 red chips, so they represent the same number.
36. Possible answer: $^{-}3 + 5$ is 3 red chips and 5 black chips, or 2 black chips; $8 + {}^{-}5$ is 8 black chips and 5 red chips, or 3 black chips; $7 + {}^{-}5$ is 7 black chips and 5 red chips, or 2 black chips; $12 + {}^{-}10$ is 12 black chips and 10 red chips, or 2 black chips; so $8 + {}^{-}5$ is different.
37. \$5.00 − \$0.75 − \$3.50 + \$2.00 − \$1.25 = \$1.50
38. **a.** $^{-}45 + 273 = 228$°K
b. $71 = C + 273$, so the temperature is $^{-}202$°C.
c. $^{-}171$°C to $^{-}43$°C

Accentuate the Negative Practice Answers

Skill: Adding Integers

1. −5 2. 5 3. 3 4. 12
5. −8 6. −3 7. −14 8. 2
9. 15 10. 5 11. −14 12. −6
13. −8 14. −26 15. −5 16. −3
17. 61 18. −25 19. 11 20. −8

Skill: Subtracting Integers

1. −17 2. −19 3. 28 4. 43
5. 27 6. 28 7. −254 8. −239
9. 51 10. −152 11. 9 12. 108
13. 57 14. −67 15. 63 16. −25
17. −7 18. 5 19. 8 20. −41

Skill: Adding and Subtracting Rational Numbers

1. $1\frac{5}{8}$ 2. $1\frac{1}{2}$ 3. $-3\frac{3}{8}$ 4. $\frac{1}{4}$
5. $\frac{31}{36}$ 6. $\frac{-9}{20}$ 7. $8\frac{11}{21}$ 8. $4\frac{113}{120}$
9. $\frac{-71}{100}$ 10. $11\frac{2}{5}$ 11. $-11\frac{7}{8}$ 12. $3\frac{5}{8}$

Skill: Coordinate Graphing

1. *L* 2. *F* 3. *D*
4. *B* 5. *I* 6. *C*
7. (−5, 0) 8. (2, 7) 9. (−2, −7)
10. (8, −8) 11. (0, 7) 12. (4, 2)
13. II 14. I 15. IV
16. III

Investigation 3 Additional Practice

1. 7 2. −3 3. −9
4. −21 5. −972 6. 8
7. −12 8. 33 9. $\frac{5}{8}$
10. −13 11. −18.72 12. $-\frac{2}{5}$
13. −8.9 14. −77.8 15. −93
16. a. $-5 \times +2 = -10$
$-10 \div -5 = +2$
$-10 \div +2 = -5$
b. $-4 \times -6 = +24$
$+24 \div -4 = -6$
$+24 \div -6 = -4$
c. $+0.6 \div -0.3 = -2$
$+0.6 \div -2 = -0.3$
$-0.3 \times -2 = +0.6$
d. $-32 \div -8 = +4$
$-32 \div +4 = -8$
$-8 \times +4 = -32$
17. Shandra won. (Figure 1)

Skill: Multiplying Integers

1. 56 2. −35 3. −32 4. 16
5. −66 6. −42 7. 64 8. 40
9. 273 10. −180 11. 350 12. −250
13. −2 14. −1 15. 2 16. −3
17. 18 18. 9 19. 48 20. −35
21. −360 22. −10 × −40; −400

Skill: Dividing Integers

1. 2 2. −7 3. −3 4. 3
5. −5 6. −7 7. 7 8. −7
9. 18 10. −12 11. −16 12. 21
13. −4 14. 13 15. 100 16. 7
17. −30 18. −11 19. −78 20. 20
21. 180 22. −\$60 per day
23. 200 feet/minute 24. −\$60 per year
25. \$2 per day

Figure 1

Juan	Shandra	Kasper
+, $^{-}2$, $^{+}3$; −6; −6	+, $^{-}1$, $^{+}2$; −2; −2	−, $^{-}6$, $^{+}1$; −6; +6
+, $^{-}4$, $^{+}1$; −4; −10	−, $^{-}5$, $^{+}2$; −10; +8	−, $^{-}1$, $^{+}4$; −4; +10
−, $^{-}2$, $^{+}2$; −4; −6	−, $^{-}3$, $^{+}4$; −12; +20	+, $^{-}1$, $^{+}4$; −4; +6
+, $^{-}3$, $^{+}6$; −18; −24	−, $^{-}3$, $^{+}5$; −15; +35	+, $^{-}1$, $^{+}5$; −5; +1
−, $^{-}2$, $^{+}6$; −12; −12	+, $^{-}6$, $^{+}6$; −36; −1	+, $^{-}4$, $^{+}4$; −16; −15

Accentuate the Negative Practice Answers

Skill: Multiplying and Dividing Rational Numbers

1. $-\frac{11}{24}$ 2. $-1\frac{1}{2}$ 3. $4\frac{3}{7}$ 4. $-\frac{67}{144}$
5. -32 6. $1\frac{13}{25}$ 7. $33\frac{29}{32}$ 8. $-1\frac{1}{10}$
9. $-30\frac{1}{4}$ 10. $-\frac{16}{225}$ 11. -4 12. $-\frac{23}{30}$
13. $7\frac{2}{3}$ 14. $-\frac{1}{4}$ 15. -63

Investigation 4 Additional Practice

1. a. -2 b. -3
   c. 1 d. 4
   e. The missing value is the opposite of the last number on the left side.
2. a. -30 b. 178
   c. $106\frac{1}{2}$ d. 32
   e. -54 f. -14
3. a. 10 b. -19
   c. -32 d. 19
   e. 12 f. 56
   g. 5 h. 24
4. a. 30 b. 40
   c. 10 d. -1.0
5. a. $-14 + 8 = -6$
   b. $\underline{1} \times -7 + \underline{1} \times 4 = \underline{-7} + 4 = \underline{-3}$
   c. $\underline{0} \times -7 + \underline{0} \times 4 = \underline{0} + \underline{0} = \underline{0}$
   d. $\underline{-1} \times -7 + \underline{-1} \times 4 = \underline{7} + \underline{-4} = \underline{3}$
   e. $\underline{-2} \times -7 + \underline{-2} \times 4 = \underline{14} + \underline{-8} = \underline{6}$
   f. When the number in front of the parentheses changes sign, the final answer changes sign. When the number in front of the parentheses is negative, the partial products have the sign opposite the numbers inside the parentheses.
6. a. $8 \times (6 + 4) = (8 \times \underline{6}) + (8 \times 4)$
   b. $7 \times (x + 3) = (7 \times \underline{x}) + (\underline{7} \times 3)$
   c. $(-9 \times 5) + (\underline{-9} \times 7) = -9 \times (\underline{5} + 7)$
   d. $(x \times 4) + (x \times 5) = \underline{x} \times (4 + 5)$
   e. $8x + 12x = x \times (\underline{8} + \underline{12})$
7. a. $-3 \cdot (4 + -7) = -3 \cdot 4 + -3 \cdot -7$
   b. $(-5 \cdot 3) - (-5 \cdot -13) = -5 \cdot (3 - -13)$
   c. $10 \cdot (-3 + 5) = 10 \cdot -3 + 10 \cdot 5$
   d. $(-12x) + (4x) = x \cdot (-12 + 4)$
   e. $2 \cdot (2 - -4) = 2 \cdot 2 - 2 \cdot -4$
   f. $(x) - (4x) = x \cdot (1 - 4)$

Skill: Order of Operations With Integers

1. 90 2. 4.75 3. 0.5 4. 38
5. 27 6. 36 7. 5 8. $8\frac{2}{3}$
9. 36 10. 4 11. 233 12. 53
13. 18 14. -18 15. 56 16. -16
17. -5 18. -29 19. -93 20. -141

Skill: Properties of Operations

1. 9; 6 2. 4; 4 3. 9; 0.2 4. 6
5. $(6 + 6) \div 6 \times (6 + 6) = 24$
6. $6 \times (6 + 6) \times 6 - 6 = 426$
7. $(6 + 6) \div 6 \times (6 - 6) = 0$
8. $(6 - 6) \times 6 + 6 \div 6 = 1$
9. $(6 + 6) \div (6 + 6) \times 6 = 6$
10. $(6 - 6) \div 6 \times (6 + 6) = 0$
11. $6h - 24$ 12. $5p + 15$
13. $-3x - 24$ 14. $-36 + 9y$
15. $14n - 22$ 16. $10a - 50$

Additional

Practice

Pages

CoordinArt Instructions

Welcome to CoordinArt and the fun practice each activity provides for plotting coordinates in all four quadrants of a Cartesian plane.

Directions:

The first number (coordinate) in every ordered pair is the *x*-coordinate. The second number is always the *y*-coordinate.

Find the *x* value along the horizontal axis and then move up or down to the appropriate *y* value. Plot the point at this intersection. Be aware there are values between the grid lines values given and some points will end up in the space between the grid lines because that's where the two values would intersect.

Using a straight edge connect the points as you go, but stop connecting when you come to the word **STOP**. Begin fresh with the next set of ordered pairs.

If a student (or you) find yourself drawing a stray line through the design you have probably either mixed up the *x* and *y* coordinates, or overlooked whether a coordinate was positive or negative.

Hint **for those new to plotting in all four quadrants:**

1) Before beginning have students take out a highlighter (or borrow one) and highlight the NEGATIVE portion of the ***x- axis***. Demonstrate for students with a document camera or using an overhead if possible.
2) Using that same highlighter....highlight ONLY the *x* coordinates on the instructions that are NEGATIVE. (Continue to demonstrate.)
3) Take a different color highlighter and highlight the NEGATIVE portion of the y- ***axis***. (Students are good at sharing with each other to find a second color to use.)
4) Using this second color highlight ONLY the negative *y*-coordinates.
5) Quickly walk around to check that coordinate grids and instructions are highlighted correctly. (Have a few extra on hand for those who didn't highlight as directed.)

Taking the time to do this helps call attention to the negative numbers and allow students more success right from the start. ☺

Happy Plotting!

Sincerely,
Nancy Coe
Creator of CoordinArt

CoordinArt Tips

QUADRANT ONE only:

- **Before beginning for the first time—**have students use a highlighter and a ruler to highlight the *x* axis. Then using the same highlighter highlight the first coordinate *(x)*in each ordered pair.

  This will help students remember the first number *(x-coordinate)* is found first along the horizontal (*x-axis).*

- **Always plot in pencil! (We ALL make mistakes. ☺)**

- **Do the first few points together with you using either an overhead projector or a document camera.**

- **Point out the "STOP's**

- **Use a straight edge to connect consecutive points within each grouping of ordered pairs.** A 6-inch ruler or protractor works great. I prefer my little 6-inch ruler to a full 12-inch one because it is small enough to quickly flip around any way I need.

- **Trouble following the correct line for the *x-coordinate* till you meet the *y-coordinate*?** Use a piece of sturdy poster board or light weight cardboard like what you find at the back of most spiral or similar notebooks. The thin white cardboard packaging that is used to keep men's dress shirts neat in the cellophane wrap works great, too. These all will allow for students to successful see both lines come together to plot the point each time.

- **WHERE WAS I?** It is important for students to keep track where they are on the coordinate sheet as they plot the points. It is best to make a little check **in front of** the ordered pair as opposed to drawing a line through each one. Drawing through the numbers prohibits students from easily looking back at them to correct a point that was incorrectly plotted. They will eventually realize if a line drawn all of a sudden goes randomly through the middle of the picture that they made a mistake in reading the coordinates.

- **COLORING TIP:** Students tend to be more motivated to make the CoordinArt when they see the colored result that I provide. But they are often afraid of ruining their result if they try to color it, so please share these tips!

  When **coloring faces** begin with a much lighter shade than you think you want. This gives you room to build depth to the face by adding darker and darker colors as needed. You can go over light colors with success, but not the other way around. Building light to dark will give 3-dimensional feel to the faces. This technique is the same whether you are coloring a Caucasian or dark, African American face. Selecting only a dark color and coloring evenly with it will result in a very flat, 2-dimensional face. This is disappointing to students after their hard work of plotting all the points!

  This same philosophy works with **coloring other subject matter** as well. If the subject matter is three-dimensional it is NOT all one even color! Where light hits the surface it reflects a lighter shade and causes shadows in other areas. Use related colors to help with shading darker. Just remember to start with the lightest colors first since you can't go light once you've gone dark.

- If students are coloring at home…suggest they use something like Google to bring up images of the subject matter. This is just to give them something to look at that will assist them seeing where the light and shadows may tend to be. Kids really do like making their pictures look as good as possible, so don't be afraid to suggest this strategy.

CoordinArt Tips

Build Awareness of All 4 Quadrants

Take the opportunity to build student awareness of all four quadrants anytime you are working with graphs and number lines.

How?

- **Whenever using horizontal number lines extend the line a little to the left. Put an arrow on the extending left end indicating there are numbers to the left of zero. What would that be? (Many will think it is fractions, so this is an important to build and reinforce awareness of negative numbers!)**
- **When talking about temperatures or elevation use a vertical number line and talk about the numbers that are below zero.**
- **When making graphs show how they are made from a vertical and a horizontal number line. Extend both of them a little with an arrow to represent the numbers less than zero on both lines (axes).**

ADDITIONAL TIPS for Plotting in All FOUR Quadrants

- Before beginning have students take out a highlighter (or borrow one) and **highlight the NEGATIVE portion of the *x*-axis.** Demonstrate for students with a document camera or using an overhead if possible.
- Using that **same highlighter....highlight ONLY the *x* coordinates** on the instructions **that are NEGATIVE.** (Continue to demonstrate.)
- Take a **different color highlighter and highlight the NEGATIVE portion of the *y*-axis.** (Students are good at sharing with each other to find a second color to use.)
- Using this second color **highlight ONLY the negative *y*-coordinates.**
- Quickly walk around to check **that coordinate grids and instructions are highlighted correctly**. (Have a few extra on hand for those who didn't highlight as directed.)

Taking the time to do this helps call attention to the negative numbers and promotes more success right from the start.

Name ______________________________ Period ______________

Integers and Absolute Value Practice

Complete.

1. $\lvert -\frac{2}{5} \rvert + 2.5$	2. $\lvert 20 \rvert$	3. $\lvert 10 \rvert$
4. $-4\frac{1}{2} + \lvert 6.3 - 7.1 \rvert$	5. $\lvert {}^{-}1 - {}^{-}20 \rvert$	6. $\lvert {}^{-}36 + 38 \rvert - \lvert {}^{-}3.5 + {}^{-}12 \rvert$
7. $\lvert 45 + {}^{-}45 \rvert$	8. $\lvert {}^{-}21 + 42 \rvert + \lvert {}^{-}13 + {}^{-}9 \rvert$	9. $\lvert {}^{-}34 - {}^{-}17 \rvert + \lvert 3^2 - 41 \rvert$
10. $\lvert {}^{-}31 - {}^{-}7 \rvert$	11. $\lvert {}^{-}43 + 25 \rvert - \lvert 3 - {}^{-}13 \rvert$	12. $\lvert {}^{-}9 - {}^{-}41 \rvert$
13. $\lvert {}^{-}34 + 3 \rvert$	14. $\lvert {}^{-}3\frac{1}{6} + {}^{-}7 \rvert - \lvert {}^{-}21 - 32 \rvert$	15. $\lvert {}^{-}29 - 26 \rvert$

16. $\|43 - {}^{-}9\| + \|{}^{-}9 - 45\|$	17. $\|{}^{-}44 - {}^{-}30\|$	18. $\|26 + {}^{-}19\|$
19. $\|{}^{-}13 - 31\| - \|14 + {}^{-}15\|$	20. $\|{}^{-}10 - {}^{-}19\| + \|{}^{-}11 - 38\|$	21. $\|6 + {}^{-}33\| + \|4 + 16\|$
22. $\|21 + {}^{-}14\|$	23. $\|{}^{-}15 + 8\|$	24. $\|{}^{-}49 + 47\| - \|{}^{-}43 + 9\|$
25. $\|{}^{-}15 + {}^{-}8\| + \|46 - 2\|$	26. $\|20 + 49\| - \|{}^{-}48 - {}^{-}30\|$	27. $\|19 + {}^{-}9\|$
28. $\|{}^{-}44 - {}^{-}2\|$	29. $\|{}^{-}31 - {}^{-}46\|$	30. $\|16 - {}^{-}14\| + \|{}^{-}15 + 29\|$

+,- Practice with Rational Numbers

Use with **ACCENTUATE THE NEGATIVE 7th**

Name____ **Period**__________

Estimate: Be sure to show how you determine your answer!

1. $14\frac{13}{20} - (-3\frac{19}{21})$

2. $8\frac{17}{24} + (-2\frac{13}{25})$

3. $-30\frac{15}{16} - 18\frac{7}{19}$

4. $(-12) - 6\frac{13}{18}$

5. $22.8 + 8.045$

6. $-11.6 - (-7.04)$

Find each sum or difference (show your work!). Write answers with fractions in simplest form.

7. $-3\frac{5}{6} - 4\frac{1}{12}$

8. $-1\frac{1}{6} + 2\frac{2}{3}$

9. $4\frac{1}{2} - (-3\frac{7}{8})$

10. $12.04 - (-6.8)$

11. $7\frac{1}{2} - 8\frac{3}{4}$

12. $-900 - (-365.6)$

13. $14.6 + (-3\frac{1}{5})$

14. $-7\frac{3}{4} + 4.125$

15. $5.75 + (-2\frac{1}{4})$

16. $-720 - 462.8$

Name ______________________ Class ______________ Date ________

Integer Puzzles

Use the given numbers to make each equation true.

3	5	7	$^-3$	$^-5$	$^-7$

1. ______ + ______ = $^-4$
2. ______ + ______ = 8
3. ______ + ______ = 2
4. ______ + ______ = 4
5. ______ + ______ = $^-12$
6. ______ + ______ = $^-2$
7. ______ + ______ = 12
8. ______ + ______ = 10
9. ______ + ______ = $^-8$
10. ______ + ______ = $^-10$

2	6	9	$^-2$	$^-6$	$^-9$

11. ______ + ______ = $^-11$
12. ______ + ______ = $^-7$
13. ______ + ______ = 4
14. ______ + ______ = $^-4$
15. ______ + ______ = 15
16. ______ + ______ = $^-15$
17. ______ + ______ = $^-8$
18. ______ + ______ = 7
19. ______ + ______ = 3
20. ______ + ______ = 11
21. ______ + ______ = 8
22. ______ + ______ = $^-3$

Make eight different addition equations using the given numbers for the addends. Then find each sum.

3	4	8	$^{-}3$	$^{-}4$	$^{-}8$

23. _____ + _____ = _____

24. _____ + _____ = _____

25. _____ + _____ = _____

26. _____ + _____ = _____

27. _____ + _____ = _____

28. _____ + _____ = _____

29. _____ + _____ = _____

30. _____ + _____ = _____

Name ______________________________ Period ______________

Multiplying and Dividing With Rational Numbers

Use with ACCENTUATE THE NEGATIVE 7th

Complete. NO CALULATORS!

1. $-121.2\ /\ 10.1$	2. $10.2 \bullet -8.3$	3. $24.36\,(-3)$
4. $-106.09 \div 10.3$	5. $\frac{22.83}{-3}$	6. 33×2.4
7. $-27 \div -5.4$	8. $162 \div 18$	9. $-94 \div 47$
10. $(-3.5)(-2)$	11. $-22 \bullet 17.7$	12. $-170 \div 10$
13. $-116.1 \div -12.9$	14. $368 \div -23$	15. $14\ /\ -7$

16. $\frac{-494}{38}$	17. $\frac{-117}{9}$	18. 18.7×32
19. -7.5×-18.6	20. $3^3 \times 1.6$	21. $\frac{2}{5} \times (-3\frac{1}{2})$
22. $-157.5 \div 9$	23. 3×6	24. $-105 \div -7$
25. $46 \bullet -7.8$	26. $\frac{2.8}{-7}$	27. $22.8 \div -5.7$
28. $30 \div 10$	29. $\frac{-18.36}{15.3}$	30. $-492 \div -41$

Name ______________________________ Date ____________ Class ____________

Additional Practice

Investigation 3

Accentuate the Negative

Find the missing value.

1. $\square \times 8 = 56$

2. $12 \times \square = {}^{-}36$

3. $\square \times {}^{-}10 = 90$

4. $7 \times \square = {}^{-}147$

5. $\square \div 18 = {}^{-}54$

6. $64 \div \square = 8$

7. ${}^{-}192 \div \square = 16$

8. ${}^{-}99.99 \div \square = {}^{-}3.03$

9. $\frac{2}{3} \times \square = \frac{10}{24}$

10. $\square \times 13 = {}^{-}169$

11. ${}^{-}234 \div 12.5 = \square$

12. $3\frac{1}{5} \div \square = -8$

13. $\square \times {}^{-}7.6 = 67.64$

14. $\square \div {}^{-}77.8 = 1$

15. $\square \div \frac{-93}{10} = 10$

Additional Practice *(continued)*

16. Write a complete fact family for each of the following:

a. $^{-}5 \times {}^{+}2 = {}^{-}10$

b. $^{-}4 \times {}^{-}6 = {}^{+}24$

c. $^{+}0.6 \div {}^{-}0.3 = {}^{-}2$

d. $^{-}32 \div {}^{-}8 = {}^{+}4$

17. For the Number Line Game there are three number cubes marked as shown:

blue number cube: +, +, +, –, –, –

red number cube: $^{-}1, {}^{-}2, {}^{-}3, {}^{-}4, {}^{-}5, {}^{-}6$

green number cube: $^{+}1, {}^{+}2, {}^{+}3, {}^{+}4, {}^{+}5, {}^{+}6$

You start with a cumulative total of 0. On each turn, you roll the cube, multiply the numbers, and add or subtract the product to your cumulative total. The winner is the person whose cumulative total is closest to 0 after each player has taken 5 turns.

Here are the rolls for Juan, Shandra, and Kasper. Who won?

Juan	Shandra	Kasper
+, $^{-}2$, $^{+}3$	+, $^{-}1$, $^{+}2$	–, $^{-}6$, $^{+}1$
+, $^{-}4$, $^{+}1$	–, $^{-}5$, $^{+}2$	–, $^{-}1$, $^{+}4$
–, $^{-}2$, $^{+}2$	–, $^{-}3$, $^{+}4$	+, $^{-}1$, $^{+}4$
+, $^{-}3$, $^{+}6$	–, $^{-}3$, $^{+}5$	+, $^{-}1$, $^{+}5$
–, $^{-}2$, $^{+}6$	+, $^{-}6$, $^{+}6$	+, $^{-}4$, $^{+}4$

Moving Straight Ahead

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations /Additional Targets
Problem 2.1, Walking to the Yogurt Shop, p. 17	1		#2 Must Do	**7.1.E Solve two-step linear equations.**
Problem 2.2, Changing the Walking Rate, p. 18	1		Must Do	
Problem 2.3, Walking for Charity, p. 19	1		Must Do	**7.1.F Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.**
Problem 2.4, Walking to Win, p. 21	1		Must Do	
Problem 2.5, Crossing the Line, p. 22	1		Must Do	
Mathematical Reflections, Investigation 2, p. 34	1			
Problem 3.1, Getting to the Point, p. 36	1		Optional	
Problem 3.2, Graphing Lines, p. 37	1		Optional	
Problem 3.3, Finding Solutions, p. 39	1		Should Do	**7.1.G Solve single- and multi step word problems involving rational numbers and verify the solutions.**
Problem 3.4, Planning a Skating Party, p. 41 and ACE Investigation 3, #29, p. 51	1		Should Do	
Proportional Relationships –teacher pg		binder		
Quiz A	1			**7.2.E Represent proportional relationships using graphs, tables, and equations and make connections among the representation.**
CMP2 Problem 3.2, Exploring Equality, p. 48-51		Binder/CMP2 Disc(7)	Must Do	
CMP2 Problem 3.3, From Pouches to Variables, p. 51-53	2	Binder/CMP2 Disc(7)	Must Do	**7.2.F Determine the slope of a line corresponding to the graph of a proportional relationship and relate slope to similar triangles.**
CMP2 Problem 3.4, Solving Linear Equations, p. 53-54	1	Binder/CMP2 Disc(7)	Must Do	
Problem 4.2, Using the Symbolic Method, p. 54, and **ACE, (Investigation 4), #1 – 9, p. 59 – 60.** (Note: Use 4.1 as your launch, there is information on the installment plan which is crucial to know to set the problem up accurately—see p. 53)	2		Must Do	**7.2.G Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.**
Problem 5.1, Climbing Stairs, p. 64 as a launch for Problem 5.2, Finding the Slope of the Line, p. 66	1		5.1 Don't Do 5.2 Must Do	Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Problem 5.3, Connecting Points, p. 68	1		Should Do	
ACE, Investigation 5, #1 – 27, p. 70-74	1			
Mathematical Reflections, Investigation 5, p. 91	1			
Looking Back and Looking Ahead Unit Reflections, p. 92 – 94	1			
Additional materials: Equation tables worksheet,Review of Linear Representations (incl. Proportional) wksht,Matching of Linear Representations	2	binder		
Moving Straight Ahead Unit Assessment	2			
Review and Reflect / Student Self-Assessment	1			
Total Instructional Days for Moving Straight Ahead:	**26**			

Contents in Moving Straight Ahead

PROPORTIONAL RELATIONSHIPS

All proportional relationships are linear, but not all linear relationships are proportional! (Kind of like the relationship between rectangles and squares.)

To be proportional looking at a table: The x/y value must make equivalent fractions

A

x	y	x/y
4	5	4/5
8	10	8/10 = 4/5
20	25	20/25 = 4/5

Yes, it is linear. Yes, it is proportional.

B

x	y	x/y
1	15	1/15
2	20	2/20=1/10
3	25	3/25

Yes, it is linear. No, it is NOT proportional.
(1/15 does not equal 2/20 or 1/10)

To be proportional looking at an equation: It can be written in the y=mx+b format AND, the y-intercept (b) is 0.

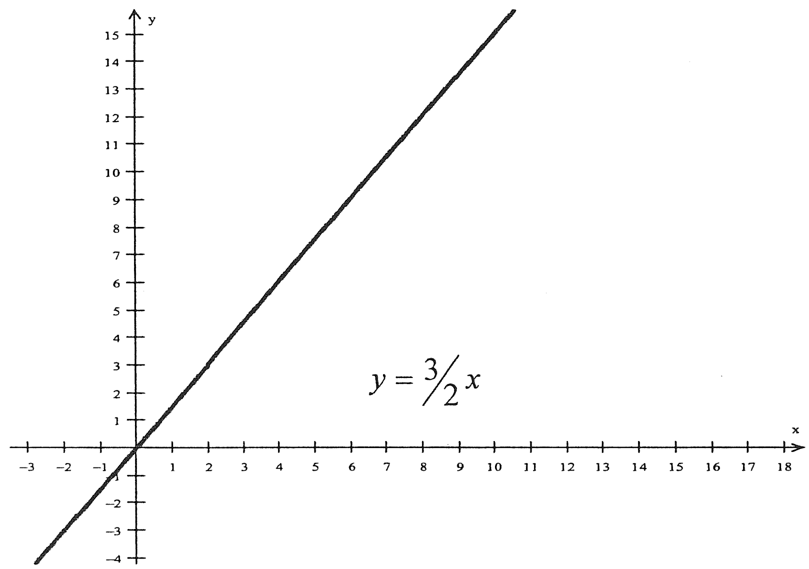
A.	B.	C.	D.	E.
y=3x+2	y=-2x (y = -2x + 0)	y= 1/2x + 0	5 + 10x (y = 10x+5)	-2/3 + x (y = x -2/3)

B and C are Proportional, and therefore, linear.
A, D, and E are linear, but NOT Proportional

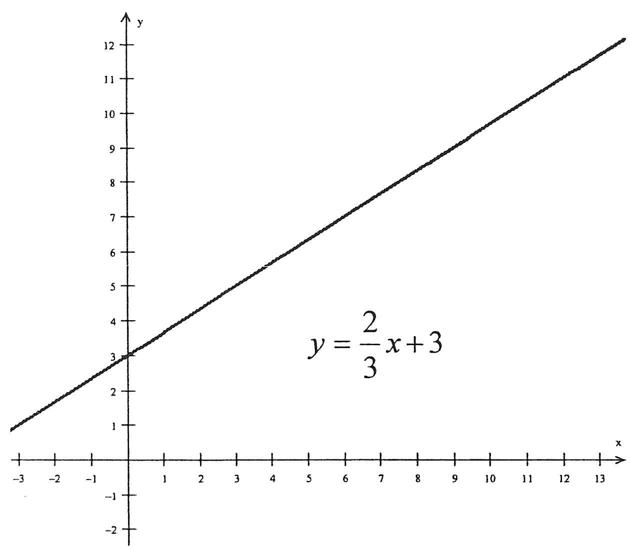
PROPORTIONAL RELATIONSHIPS

To be proportional looking at a graph: The straight line MUST pass through the point of origin (0, 0).

The following graph is linear AND it is proportional.



The following graph is linear, but it is NOT proportional.



Problem 3.1 Solving Equations Using Tables and Graphs

A. Use the equation $A = 5 + 0.5d$.

1. Suppose Alana walks 23 kilometers. Show how you can use a table and a graph to find the amount of money Alana gets from each sponsor.

2. Suppose Alana receives \$60 from a sponsor. Show how you can use a table and a graph to find the number of kilometers she walks.

B. For each equation:

- Tell what information Alana is looking for.
- Describe how you can find the information.

1. $A = 5 + 0.5(15)$

2. $50 = 5 + 0.5d$

C. The following equations are related to situations that you have explored. Find the solution (the value of the variable) for each equation. Then, describe another way you can find the solution.

1. $D = 25 + 2.5(7)$

2. $70 = 25 + 2.5t$

ACE Homework starts on page 57.

3.2 Exploring Equality

An equation states that two quantities are equal. In the equation $A = 5 + 0.5d$, A and $5 + 0.5d$ are the two quantities. Both represent the amount of money that Alana collects from each sponsor. Since each quantity represents numbers, you can use the properties of numbers to solve equations with one unknown variable.

Before we begin to solve linear equations, we need to look more closely at equality.

What does it mean for two quantities to be equal?

Let's look first at numerical statements.

Getting Ready for Problem 3.2

The equation $85 = 70 + 15$ states that the quantities 85 and $70 + 15$ are equal.

What do you have to do to maintain equality if you

- subtract 15 from the left-hand side of the equation?
- add 10 to the right-hand side of the original equation?
- divide the left-hand side of the original equation by 5?
- multiply the right-hand side of the original equation by 4?

Try your methods on another example of equality. Summarize what you know about maintaining equality between two quantities.

In the Kingdom of Montarek, the ambassadors carry diplomatic pouches. The contents of the pouches are unknown except by the ambassadors. Ambassador Milton wants to send one-dollar gold coins to another country.

$1 gold coin

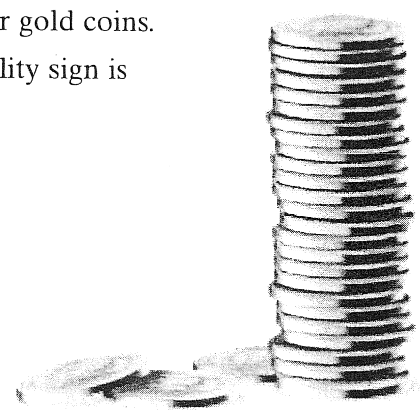
diplomatic pouch

His daughter, Sarah, is a mathematician. She helps him devise a plan based on *equality* to keep track of the number of one-dollar gold coins in each pouch.

In each situation:

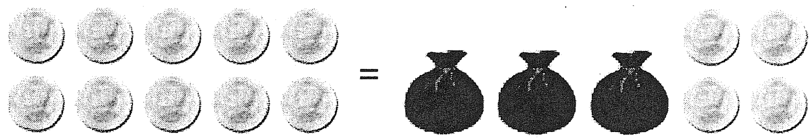
- Each pouch contains the same number of one-dollar gold coins.
- The number of gold coins on both sides of the equality sign is the same, but some coins are hidden in the pouches.

Try to find the number of gold coins in each pouch.



Problem 3.2 Exploring Equality

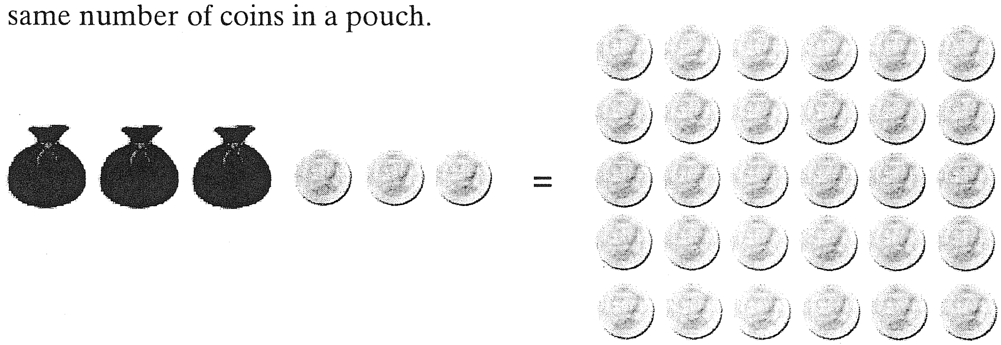
A. Sarah draws the following picture. Each pouch contains the same number of $1 gold coins.



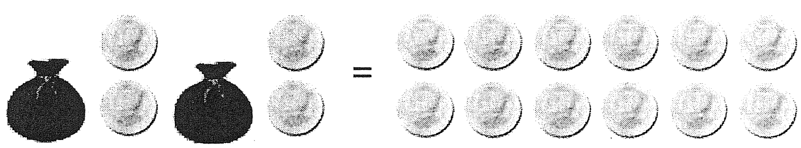
How many gold coins are in each pouch? Explain your reasoning.

B. For each situation, find the number of gold coins in the pouch. Write down your steps so that someone else could follow your steps to find the same number of coins in a pouch.

1.

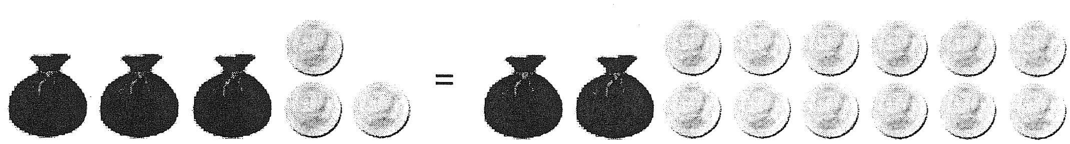


2.

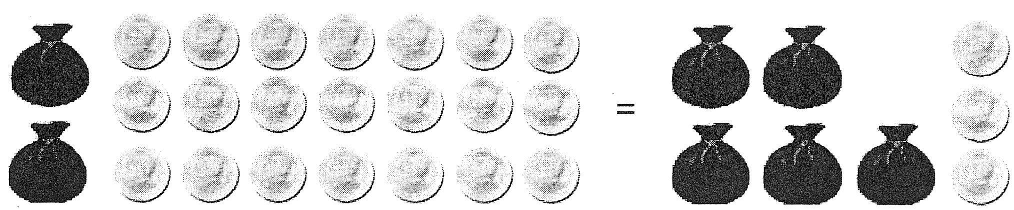


3.

4.



5.



C. Describe how you can check your answer. That is, how do you know you found the correct number of gold coins in each pouch?

D. Describe how you maintained equality at each step of your solutions in Questions A and B.

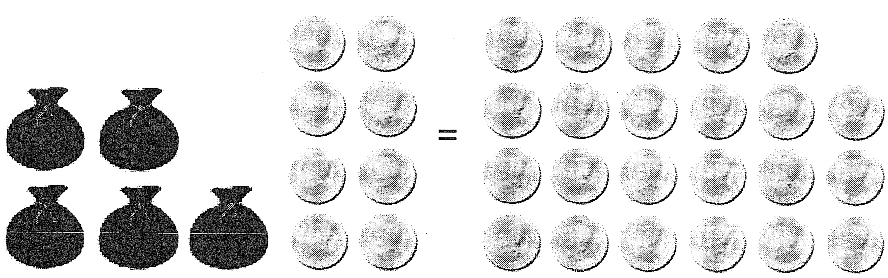
ACE **Homework starts on page 57.**

3.3 From Pouches to Variables

Throughout this unit, you have been solving problems that involve two variables. Sometimes the value of one variable is known, and you want to find the value of the other variable. The next problem continues the search for finding a value for a variable without using a table or graph. In this investigation, you are learning to use *symbolic* methods to solve a linear equation.

Getting Ready for Problem 3.3

The picture below represents another diplomatic pouch situation.



Because the number of gold coins in each pouch is unknown, we can let x represent the number of coins in one pouch and 1 represent the value of one gold coin.

- Write an equation to represent this situation.
- Use your methods from Problem 3.2 to find the number of gold coins in each pouch.
- Next to your work, write down a similar method using the equation that represents this situation.

3.2 Exploring Equality

Goals

- Develop understanding of equality
- Begin to use the properties of equality to solve equations that are represented pictorially

The properties of equality are introduced in a Getting Ready prior to Problem 3.2. Students use these properties informally to find the number of coins in a pouch. Equations are represented pictorially as coins (constant term) and pouches (variables).

Mathematics Background

For background on the properties of equality, see pages 6–7.

Launch 3.2

Discuss the Getting Ready.

Write the statement: $85 = 70 + 15$.

Suggested Questions Ask:

- *Is this a true statement?* (Yes. 85 and $70 + 15$ are equivalent ways to write the same quantity.)

Now pose each of the questions one at a time, always using the original equation. For example, for the first statement, you could record the following:

$$\begin{array}{rcl} 85 &=& 70 + 15 \\ -15 &=& \quad -15 \\ \hline 70 &=& 70 \end{array} \qquad \text{OR} \qquad \begin{array}{rcl} 85 &=& 70 + 15 \\ 85 - 15 &=& 70 + 15 - 15 \\ 70 &=& 70 \end{array}$$

After subtracting 15 from the left side of the equation, you could ask,

- *What should we do to keep the two sides equal?*

Repeat this method for each of the statements in the Getting Ready.

You could pose another problem and repeat the questions. For example:

- *Do your methods work on the following equality: $64 = 24 + 4(10)$?*
- *Can you start by dividing each side by 4? Do we still have a true statement?* (Dividing by 4 will maintain the balance. Intuitively, this makes sense, but students may make the error of dividing only one part of the right side expression, perhaps producing the result $16 = 6 + 4(10)$ or $16 = 24 + 10$, neither of which is true. If we divide the total on both sides by 4 we get $16 = 6 + 10$, which is true.)
- *Can we subtract 24 from each side and still have equality?*
- *Is the resulting $40 = 4(10)$ easier to divide? Why?* (There is usually more than one of the properties of equalities that can be used as a first step in solving equations, but some properties are easier to accomplish than others.)
- *Summarize what you know about maintaining equality.* (You can add or subtract both sides of an equality by the same number and maintain equality. You can also multiply or divide by a non-zero number and have equality.)
- *These are called the principles of equality.*

To launch Problem 3.2, tell the class about the diplomatic pouches and gold coins in the Kingdom of Montarek:

- *In each situation, each pouch contains the same number of one- dollar gold coins.*
- *The number of gold coins on both sides of the equality sign is the same, but some coins are hidden in the pouches.*
- *Your challenge is to find the number of coins in each pouch.*

You might want to have students work on Question A and then summarize it or you can do it as a whole class activity.

As you discuss their strategies for Question A, you can script their work as you go (Figure 1).

Let students work in pairs.

Explore 3.2

Encourage students to make a record of their strategies. They may want to put their work on large poster paper.

Suggested Questions As you move around, ask:

- *What does equality mean?*
- *How can we maintain equality?*
- *How do we know that our answer is correct?* (This is an opportunity to see how students handle parentheses. See answers for some suggestions.)

Look for interesting strategies to share in the summary.

Summarize 3.2

Have different pairs of students discuss their work for a problem. Be sure that they show how they checked their answer. The following is an example of how one group of students solved Question A.

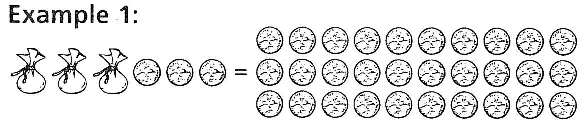
> First we noted that there are 10 coins on the left and there are 4 coins and 3 bags on the right. So, the three bags must have a total of 6 coins to make the total number of coins on the right equal 10. Each bag must have 2 coins.
>
> Take four coins off of each side. Now we have 6 coins on the left and 3 bags on the right. Each bag must have 2 coins.

Suggested Questions After all the parts have been discussed, ask:

- *Did each group use the same first step in each problem?* (As was discussed in the launch, there is often more than one way to make the first step in solving an equation.)
- *What are some common strategies that we can use to maintain equality?* (As students describe the strategy, ask them to refer to a step in a problem solution that illustrates the strategy. You could put the properties of equality on large poster paper.)

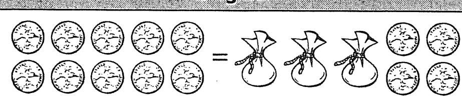
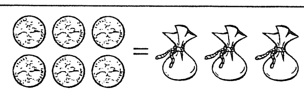
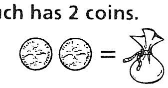
Use one of the situations from Question B to make the transition from pouches to variables. This could serve as a launch to Problem 3.3.

Example 1:

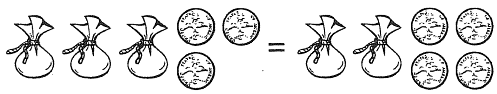


- *Suppose we let x represent the number of coins in a pouch and 1 represent one coin. Rewrite the equality using x's and numbers.* ($3x + 3 = 30$)
- *Write the steps for solving this equation for x.* (Some students may subtract 3 from both sides. Some may start by dividing both sides by 3. Some may use a fact family idea and rewrite $3x + 3 = 30$ as $3x = 30 - 3$. Continue with each method to show that they are equivalent.)
- *Check your solution.*

Figure 1

Original	Strategies
	
	Take four coins from both sides of the equation.
So each pouch has 2 coins. 	Divide both sides by 3.
There are 10 coins on each side of the equality.	Check back in the original problem.

Example 2:
Find the number of coins in a pouch for the following situation:



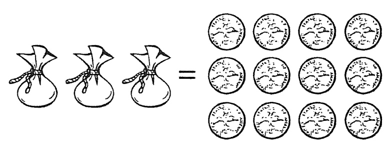
Ask the class to solve the problem visually and then by using properties of equality. Write the steps as you go along. You could add a fourth column and write in the property. When you are done, ask if there was a different first step that could be taken. Follow it through to the end (Figure 2).

Be sure to emphasize that in a given equation, the value of x or the number of coins in each pouch is the same. Checking the answer provides an opportunity for students to practice work with parentheses and review the distributive property. In the preceding example to check the answers, students could write $3(1) + 3 = 2(1) + 4$ or $6 = 6$. They could also write $1 + 1 + 1 + 3 = 1 + 1 + 4$. Review the use of parentheses if necessary.

An opportunity to revisit and apply the Distributive Property occurs in Question B part (3). Students could reason as follows:

> In Question B part (2), you could divide both sides by 2 (split each side evenly into two groups of 1 pouch and 2 coins on one side and two groups of 6 coins on the other) so you get one pouch and 2 coins on the left and 6 coins on the right. Symbolically, $2(x + 2) = 12$ becomes $x + 2 = 6$. Next, subtract 2 coins from each side to get one pouch equals 4 coins. So there must be 4 coins in each bag. Symbolically, $x + 2 = 6$ becomes $x = 4$. Students could first apply the distributive property to the left side to get 2 pouches and 4 coins on the left. Symbolically, $2(x + 2) = 12$ becomes $2x + 4 = 12$ and so on. Next, they could subtract 4 coins from each side to have 2 pouches on the left and 8 coins on the right. Next, they divide both sides by two, which leaves one pouch on the left and 4 coins on the right so there are 4 coins in each pouch. And to check our work: $2(4 + 2) = 12$ or $12 = 12$ or $(4 + 2) + (4 + 2) = 12$.

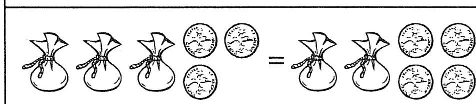
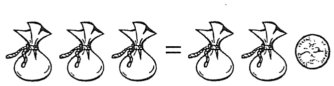
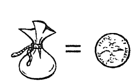
An opportunity to discuss a common misunderstanding arises with this context. Ask students to consider the equality $3x = 12$. Suppose a student proposes that subtracting 3 from both sides will produce $x = 9$. Ask students to use the idea of gold coins in pouches to investigate this strategy. (Moving back to the visual



makes it clear that dividing, not subtracting, is the logical operation.)

The conditions under which the equality relation is maintained are called the *properties of equality*. Note that this is not essential vocabulary, but it might help in the discussion of solving equations.

Figure 2

Diplomatic Pouch Problem		
	Steps for finding the number of coins in a pouch.	**Symbolic representation: One pouch is x. A coin has a value of 1.**
		$3x + 3 = 2x + 4$
	Subtract 3 coins from both sides of the equality.	$3x = 2x + 1$
	Subtract 2 pouches from both sides of the equality. Each pouch has 1 coin.	$x = 1$
There are a total of 6 coins on each side of the original equation.		Check your answer.

3.2 Exploring Equality

At a Glance

PACING 1 day

Mathematical Goal

- Develop understanding of equality
- Begin to use the properties of equality to solve equations that are represented pictorially

Launch

Discuss the Getting Ready. Write the statement: $85 = 70 + 15$. Ask if this is a true statement and why. Now pose each of the questions in the Getting Ready. one at a time, always using the original equation. You could pose another problem for example:

- *Do your methods work on the following equality:* $64 = 24 + 4(10)$*?*
- *Can you start by dividing each side by 4? Do we still have a true statement?*
- *Can we subtract 24 from each side and still have equality?*
- *Is the resulting* $40 = 4(10)$ *easier to divide? Why?*

Summarize what you know about maintaining equality.

You might want to have students work on Question A and then summarize it or you can do it as a whole class activity. As you discuss their strategies, you can script their work as you go. See the extended section for an example of a script. Let students work in pairs.

Materials
- Large sheets of poster paper
- Labsheet 3.2
- Transparency 3.2A

Explore

Encourage students to make a record of their strategies. They may want to put their work on large poster paper. Consider asking:

- *What does equality mean? How can we maintain equality?*
- *How do we know that our answer is correct?*

Summarize

Have student pairs discuss their work for a problem. Ask how they checked their answer. After all the parts have been discussed, ask:

- *Did each group use the same first step in each problem?*
- *What are some common strategies that we can use to maintain equality?*

Use one of the situations from Question B to make the transition from pouches to variables. For Question B part (1), for example:

Suppose we let x represent the number of coins in a pouch and 1 represent one coin. Rewrite the equality using x's and numbers.

- *Write the steps for solving this equation for x. Check your solution.*

Emphasize that in a given equation, the value of x or the number of coins in each pouch is the same. Checking the answer provides an opportunity for students to practice work with parentheses and review the Distributive Property. Another opportunity to revisit and apply the Distributive

Materials
- Student notebooks
- Transparency 3.2B

continued on next page

Summarize

continued

Property occurs in Question B, part (3). See the extended section for possible student strategies. Be sure to discuss the common error of attempting to subtract, rather than divide, to eliminate the coefficient of x. See extended sections. The conditions under which the equality relation is maintained are called the properties of equality. Note that this is not essential vocabulary, but it might be useful in discussion.

ACE Assignment Guide for Problem 3.2

Differentiated Instruction
Solutions for All Learners

Core 5–8

Other *Applications* 9; *Connections* 31–33, 38; unassigned choices from previous problems

Adapted For suggestions about adapting other ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 31, 32: *Accentuate the Negative*; 33: *Bits and Pieces*

Answers to Problem 3.2

A. 2 coins per pouch; Because there are 4 coins on the right side, you can take those four away if you take four away from the left side. So now you have 6 coins on the left side and 3 pouches on the right side. There must be 2 coins in each pouch since 3 pouches with 2 coins in a pouch gives you 6 coins.

Check: $10 = 3(2) + 4$ or $10 = 6 + 4$.

B. **1.** 9 coins. Check: $3(9) + 3 = 27 + 3 = 30$

2. 4 coins. Check: $2(4) + 4 = 12$ or $12 = 12$

3. 12 coins. Check: $3(12) = 2(12) + 12$ or $36 = 36$.

4. 9 coins. Check: $3(9) + 3 = 2(9) + 12$ or $30 = 30$.

5. 6 coins. Check: $2(6) + 21 = 5(6) + 3$ or $33 = 33$

Possible steps for each part of Question B

In B1, take away 3 coins from each side so that you are left with 3 pouches on the left side and 27 coins on the right. Now split 27 evenly into 3 pouches, so each pouch must contain 9 coins.

In B2, you could divide both sides by 2 (split each side evenly into two groups of 1 pouch and 2 coins on one side and two groups of 6 coins on the other) so you get one pouch and 2 coins on the left and 6 coins on the right. Next, subtract 2 coins from each side to get one pouch equals 4 coins. So there must be 4 coins in each bag. Students could first apply the distributive property to the left side to get 2 pouches and 4 coins on the left. Next, they could subtract 4 coins from each side to have 2 pouches on the left and 8 coins on the right. Next, they divide both sides by two, which leaves one pouch on the left and 4 coins on the right. So there are 4 coins in each pouch.

In B3, you could subtract two pouches from both sides, which leaves one pouch on the left and 12 coins on the right. So each pouch has 12 coins.

In B4, take away 3 coins from each side so that you're left with 3 pouches on the left side and 2 pouches and 9 coins on the right side. Now take away two pouches from both sides and you are left with 1 pouch on the left side and 9 coins on the right side. So a pouch must contain 9 coins.

In B5, take away 3 coins from each side and two pouches from each side. You are left with 18 coins on the left side and 3 pouches on the right side. Now you must split 18 coins evenly into 3 pouches, so each pouch must contain 6 coins.

C. You could add the number of coins in each bag and on the outside of the bags on each side of the equality. If the total number on each side should be the same, then the solution is correct.

D. To maintain equality, you can add, subtract, multiply, or divide by the same number on each side of the equation.

Note Students may blend fact-family ideas and properties of equality in logical strategies. For example, in B5 students may say they should subtract 3 coins and 2 pouches from each side, leaving 18 coins = 3 pouches. They may think

$3(?) = 18$ and answer 1 pouch $= \frac{18}{3}$ coins.

C. Describe how you can check your answer. That is, how do you know you found the correct number of gold coins in each pouch?

D. Describe how you maintained equality at each step of your solutions in Questions A and B.

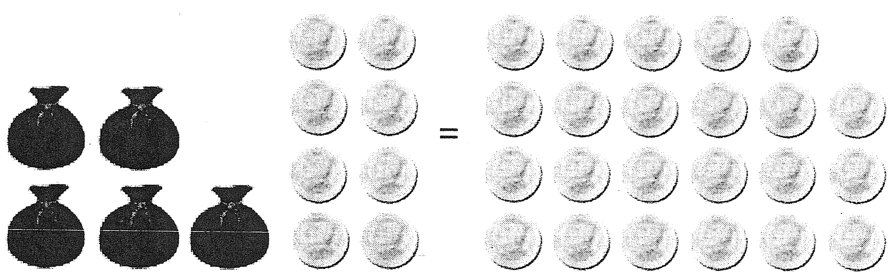
ACE **Homework starts on page 57.**

3.3 From Pouches to Variables

Throughout this unit, you have been solving problems that involve two variables. Sometimes the value of one variable is known, and you want to find the value of the other variable. The next problem continues the search for finding a value for a variable without using a table or graph. In this investigation, you are learning to use *symbolic* methods to solve a linear equation.

Getting Ready for Problem 3.3

The picture below represents another diplomatic pouch situation.



Because the number of gold coins in each pouch is unknown, we can let x represent the number of coins in one pouch and 1 represent the value of one gold coin.

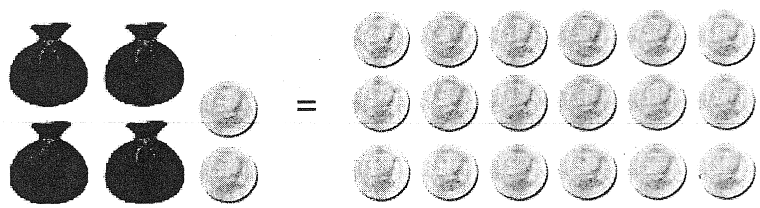
- Write an equation to represent this situation.
- Use your methods from Problem 3.2 to find the number of gold coins in each pouch.
- Next to your work, write down a similar method using the equation that represents this situation.

Problem 3.3 Writing Equations

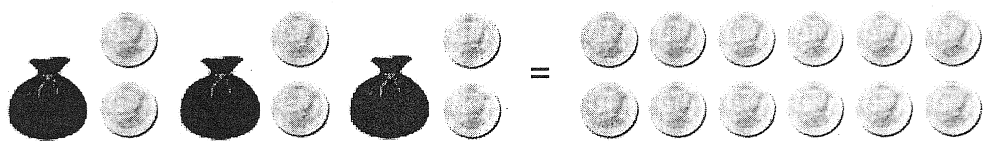
A. For each situation:

- Represent the situation with an equation. Use an x to represent the number of gold coins in each pouch and a number to represent the number of coins on each side.
- Use the equation to find the number of gold coins in each pouch.

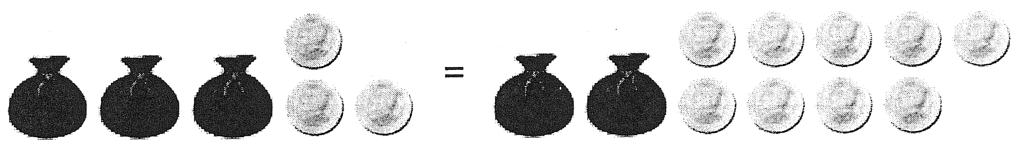
1.



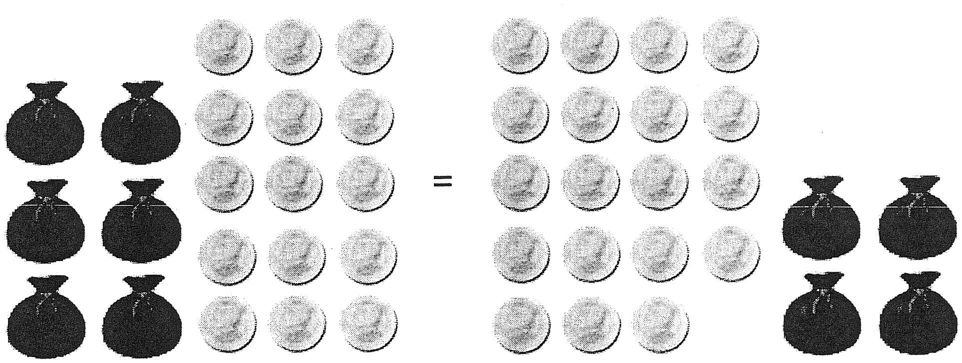
2.



3.



4.



B. For each equation:

- Use your ideas from Question A to solve the equation.
- Check your answer.

1. $30 = 6 + 4x$

2. $7x = 5 + 5x$

3. $7x + 2 = 12 + 5x$

4. $2(x + 4) = 16$

C. Describe a general method for solving equations using what you know about equality.

ACE **Homework starts on page 57.**

3.4 Solving Linear Equations

You know that to maintain an equality, you can add, subtract, multiply, or divide both sides of the equality by the same number. These are called the **properties of equality.** In the last problem, you applied properties of equality and numbers to find a solution to an equation.

So far in this investigation, all of the situations have involved positive numbers.

Does it make sense to think about negative numbers in a coin situation?

Getting Ready for Problem 3.4

- How do these two equations compare?

$2x + 10 = 16$ $\qquad$ $2x - 10 = 16$

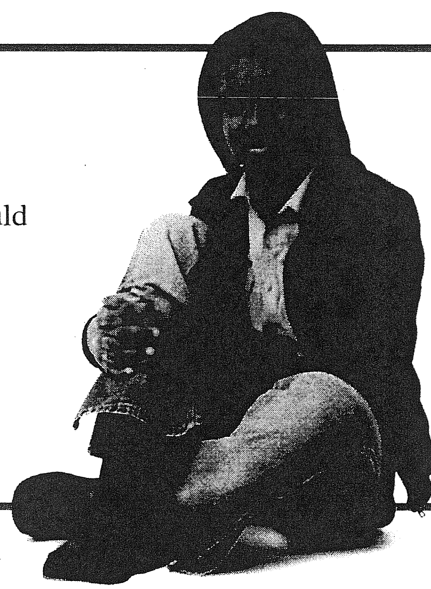
How would you solve each equation? That is, how would you find a value of x that makes each statement true?

- How do the equations below compare?

$3x = 15$ $\qquad$ $-3x = 15$

$3x = -15$ $\qquad$ $-3x = -15$

Find a value of x that makes each statement true.



3.3 From Pouches to Variables

Goals

- Use the properties of equality to solve equations
- Check solutions to equations

Launch 3.3

You can use the summary from Problem 3.2 to launch this problem. If you did not make the transition from pouches and coins to variables and numbers, use the Getting Ready as a launch or another check to see if students have a general idea of the procedures.

Suggested Questions Ask:

- *Write an equation to represent the situation in the Getting Ready.* ($5x + 8 = 23$)
- *Use your methods from Problem 3.2 to find the number of gold coins in each pouch.* (The first step is to take away 8 coins from both sides. That leaves 5 pouches on the left side and 15 coins on the right side. The second step is to divide the 15 coins into 5 equal parts to get that the number of coins in each pouch is 3.)
- *Next to your work, write down a similar method using the equation that represents this situation.* Step 1: $5x + 8 - 8 = 23 - 8$ so $5x = 15$. For Step 2, we would divide both sides of the equation by 5 and get $x = 3$.

Students should be able to generalize that adding, subtracting, multiplying, and dividing by the same number maintains equality. Be sure to stress that we cannot multiply or divide by 0. Multiplying by 0 would make any statement always true and so is not a helpful strategy in finding a specific solution. For example, suppose we start with $3b = 27$, which we know is true only when $b = 9$, then multiply by 0 which gives $3b \times 0 = 27 \times 0$ or $0 = 0$, which is always true, not just for $b = 9$. So multiplying both sides by 0 gives an equality that is not equivalent to the original equation because it has different solutions. Dividing by zero has no meaning.

If you have not already addressed the common error of subtracting when dividing is needed (For example, starting with $3x = 9$, why is dividing by 3 on both sides the logical way to produce $x = 3$, whereas, subtracting 3 to produce $x = 6$ is not logical?) do so now.

Let the class work in pairs.

Explore 3.3

Be sure students are recording their methods and checking their answers.

See how students are translating the situation in Question A part (2). Students can write it as $(x + 2) + (x + 2) + (x + 2)$ or as $3(x + 2)$ on the left side and 12 on the right. Return to this in the summary and ask if the two expressions are equivalent and why. This will review the Distributive Property that was developed in the *Accentuate the Negative* unit.

Again, poster paper or transparencies can be used to record some of the student work.

Summarize 3.3

Post the solutions around the room. Call on different students to describe how they solved the equations. Make sure you highlight problems in which students may have done something different for the first step. Some may work first with combining pouches, others will work first by working on the coins.

Suggested Questions Ask:

- *What property of equality was used at each step?*
- *Is there another way to make the first step in the solution? Explain.* (In Question B part (3), some students may start by subtracting 2 or 12 from both sides of the equation. Others may start by subtracting 5 pouches or 7 pouches (or $5x$ or $7x$) from each side of the equation. Note that some of these strategies will have students working with negative numbers. Again, this is an opportunity to check on their understanding and use of integers.)
- *Are there other equations where a different first step could be taken? Explain.*

- *What is different about the solution to equation 2 in Question B part (2)?* (It has a non-whole-number solution.)
- *Does this have any meaning in the Kingdom of Montarek?* ($x = 2.5$ could mean that you have $2.5 dollars or 2 gold $1 coins and 1 fifty-cent piece.)
- *Look back over your work for each equation. What general rules were you using to solve the equations?*
- *Describe a general method for solving equations.* (Generally, we want to isolate the variable (pouch) on one side of the equation and a number (coins) on the other side. To do this, we can use properties of equality to isolate the variables or undo the operations until we have the variable alone on one side and a number on the other side of the equality. We use this number that the variable equals to check our solution. We substitute the number for the value of the variable in the original equation to see if we have a true statement.)

3.3 From Pouches to Variables

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Use the properties of equality to solve equations
- Check solutions to equations

Launch

You can use the summary from Problem 3.2 to launch this problem. If you did not make the transition from pouches and coins to variables and numbers, use the Getting Ready as a launch or another check to see if students have a general idea of the procedures. Students should be able to generalize that adding, subtracting, multiplying, and dividing by the same number maintains equality. Be sure to stress that we cannot multiply or divide by 0, since multiplying by 0 would make any statement always true. Dividing by zero has no meaning. Let the class work in pairs.

Materials

- Large sheets of poster paper
- Transparencies 3.3A and 3.3B

Explore

Be sure students are recording their methods and checking their answers. See how students are translating the situation in Question A part (2). Students can write it as $(x + 2) + (x + 2) + (x + 2)$ or as $3(x + 2)$ on the left side and 12 on the right. Return to this in the summary and ask if the two expressions are equivalent and why. This will review the Distributive Property that was developed in the *Accentuate the Negative* unit.

Materials

- Labsheet 3.3

Summarize

Post the solutions around the room. Call on different students to describe how they solved the equations. Consider asking:

- *What property of equality was used at each step?*
- *Is there another way to make the first step in the solution? Explain.*
- *Are there other equations in which a different first step could be taken? Explain.*
- *What is different about the solution to equation 2 in Question B2?*
- *Does this have any meaning in the Kingdom of Montarek?*
- *Look back over your work for each equation. What general rules were you using to solve the equations?*
- *Describe a general method for solving equations.*

Generally, we want to isolate the variable (number of coins in a pouch) on one side of the equation and a number (coins) on the other side. To do this, we can use properties of equality to isolate the variables or undo the operations until we have the variable alone on one side and a number on the other side of the equality. We use this number that the variable equals to check our solution. We substitute the number for the value of the variable in the original equation to see if we have a true statement.

Materials

- Student notebooks

ACE Assignment Guide for Problem 3.3

Differentiated Instruction
Solutions for All Learners

Core 10, 12–14, 39, 40, 43

Other *Applications* 11; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 3.3

A. 1. $4x + 2 = 18$

$4x = 16$ (Subtract two from both sides of the equation. You can also start by dividing by 2.)

$x = 4$ (Divide both sides by 4.)

Check:

$4(4) + 2 \stackrel{?}{=} 18$

$18 = 18$

2. $x + 2 + x + 2 + x + 2 = 12$ or

$3(x + 2) = 12$ or

$3x + 6 = 12$

$x + 2 = 4$

(Divide both sides by 3. Or, apply the distributive property and then divide or subtract.)

$x = 2$

(Subtract 2 from both sides)

Check:

$3(2) + 6 \stackrel{?}{=} 12$

$12 = 12$

3. $3x + 3 = 2x + 9$

$x + 3 = 9$ (Subtract $2x$ from both sides. You can also start by subtracting 3 from both sides.)

$x = 6$ (Subtract 3 from both sides)

Check:

$3(6) + 3 \stackrel{?}{=} 2(6) + 9$

$21 = 21$

4. $6x + 15 = 4x + 19$

$6x = 4x + 4$ (Subtract 15 from both sides.)

$2x = 4$ (Subtract $4x$ from both sides.)

$x = 2$ (Divide both sides by 2.)

Check:

$6(2) + 15 \stackrel{?}{=} 4(2) + 19$

$27 = 27$

B. 1. $30 = 6 + 4x$

$24 = 4x$

$6 = x$

Check:

$30 \stackrel{?}{=} 6 + 4(6)$

$30 = 30$

2. $7x = 5 + 5x$

$2x = 5$

$x = 2.5$

Check:

$7(2.5) \stackrel{?}{=} 5 + 5(2.5)$

$17.5 = 17.5$

3. $7x + 2 = 12 + 5x$

$2x + 2 = 12$

$2x = 10$

$x = 5$

Check:

$7(5) + 2 \stackrel{?}{=} 12 + 5(5)$

$37 = 37$

4. $2(x + 4) = 16$

$x + 4 = 8$

$x = 4$

Check:

$2(4 + 4) \stackrel{?}{=} 16$

$16 = 16$

C. To find a solution to a linear equation, apply the principles of equality repeatedly to the equation until you have isolated the variable to one side of the equation and a number on the other side. Then you must check the solution by substituting the value of the variable into the original equation.

B. For each equation:

- Use your ideas from Question A to solve the equation.
- Check your answer.

1. $30 = 6 + 4x$

2. $7x = 5 + 5x$

3. $7x + 2 = 12 + 5x$

4. $2(x + 4) = 16$

C. Describe a general method for solving equations using what you know about equality.

ACE **Homework starts on page 57.**

3.4 Solving Linear Equations

You know that to maintain an equality, you can add, subtract, multiply, or divide both sides of the equality by the same number. These are called the **properties of equality.** In the last problem, you applied properties of equality and numbers to find a solution to an equation.

So far in this investigation, all of the situations have involved positive numbers.

Does it make sense to think about negative numbers in a coin situation?

Getting Ready for Problem 3.4

- How do these two equations compare?

$2x + 10 = 16$ $\qquad$ $2x - 10 = 16$

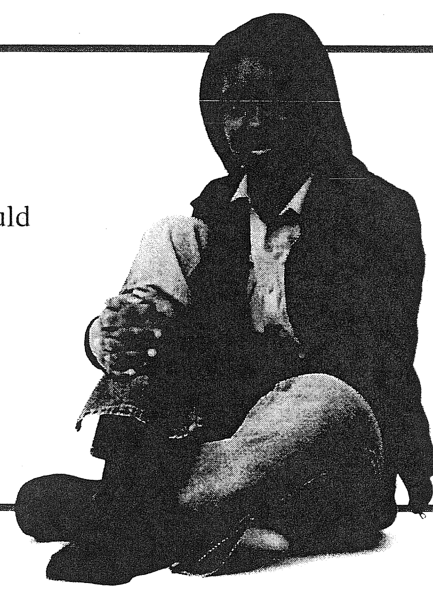
How would you solve each equation? That is, how would you find a value of x that makes each statement true?

- How do the equations below compare?

$3x = 15$ $\qquad$ $-3x = 15$

$3x = -15$ $\qquad$ $-3x = -15$

Find a value of x that makes each statement true.



Problem 3.4 Solving Linear Equations

Use what you have learned in this investigation to solve each equation.

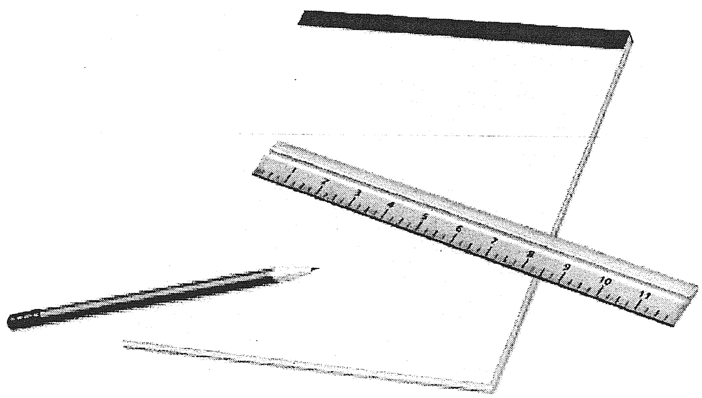
For Questions A–D, record each step you take to find your solution and check your answer.

A. **1.** $5x + 10 = 20$

2. $5x - 10 = 20$

3. $5x + 10 = -20$

4. $5x - 10 = -20$



B. **1.** $10 - 5x = 20$

2. $10 - 5x = -20$

C. **1.** $4x + 9 = 7x$

2. $4x + 9 = 7x + 3$

3. $4x - 9 = 7x$

4. $4x - 9 = -7x + 13$

D. **1.** $3(x + 2) = 21$

2. $-3(x - 5) = 2x$

3. $5(x + 2) = 6x + 3$

E. In all of the equations in Questions A–D, the value of x was an integer, but the solution to an equation can be any real number. Solve the equations below, and check your answers.

1. $5x + 10 = 19$

2. $5x + 10 = 9x$

3. $5x - 10 = -19$

4. $5x - 10 = -7x + 1$

F. **1.** Describe how you could use a graph or table to solve the equation $5x + 10 = -20$.

2. Suppose you use a different letter or symbol to represent the value of the unknown variable. For example, $5n + 10 = 6n$ instead of $5x + 10 = 6x$.

Does this make a difference in solving the equation? Explain.

ACE **Homework starts on page 57.**

3.4 Solving Linear Equations

Goals

- Develop strategies for solving linear equations
- Recognize subtle shifts in equations and how they affect the solution.

Launch 3.4

Put up the first two equations from the Getting Ready:

$2x + 10 = 16 \qquad 2x - 10 = 16$

- *Compare these two equations.* (They are the same except that 10 is added to left side of the first equation and 10 is subtracted from the left side of the second equation.)

Suggested Questions Ask:

- *How would you solve these two equations?* (In the first equation you would subtract 10 from both sides of the equation since we want to "undo" the "+10". In the second equation you would add 10 to both sides since we want to "undo' the "−10". Some students may note that you could divide by 2 first.)

The following is a quotation from a teacher: "At this point students have discovered and accepted the algorithm of 'undoing' the order of operations. The ruba model just reveals the algorithm and then they go from there."

- *In the first equation, rather than subtract 10 from both sides, could we add −10 to both sides?* (Yes, because subtracting 10 is the same as adding a −10. Refer back to *Accentuate the Negative* if students need more help with this.)

For the second equation, $2x - 10 = 16$, ask:

- *How can you solve this equation?* (One student responded that it is like you owe 10 coins. Add 10 coins to each side. Since $2x = 26$, $x = 13$. Some students have commented that this is similar to red chips in *Accentuate the Negative* unit.)
- *Compare these four equations.* (They are all of the form $ax = b$. They represent the product of 2 numbers.)
- *What does the 3x mean?* (It means that the value of x is multiplied by 3 or we are adding x 3 times. $3x = 3$ times x or $3x = x + x + x$.)

Repeat this question for $-3x$.

- *How would you solve each equation?* (Since there is no constant term on the left or a variable on the right, you divide each side by the coefficient of x. The answer will be either 5 or −5, depending on whether the division involves two positive numbers, two negative numbers, or a positive and a negative number.)

To help students focus on individual parts of the equations, ask:

- *In Question A, how are the equations alike and how are they different?*

Students can work on the problems in pairs and then move into larger groups of 4 to discuss their work.

Explore 3.4

Suggested Question Ask students to look at the four equations in Question A.

- *What do these equations have in common?*

Be sure students are making a record of their methods and checking their answers.

Encourage students to look for commonalities and differences in each set of equations. This should help them make the shift from equations that had all positive numbers to those that also include negative numbers.

Suggested Questions If students are puzzled as to how to proceed, ask questions like the following:

- *What operations on x have been used to create the expressions on each side?*
- *Look at the equation. What is a good starting step?*
- *How can you get rid of or "undo" the operation, but still maintain equality?*

You could have different groups put up solutions to different parts, A, B, C, D, and E, on poster paper to refer to in the summary.

Summarize 3.4

Go over each part of the problem. Call on different students to discuss their methods.

Suggested Questions After a group has presented their solutions, ask the class if they have any questions they want to ask the group. For example, encourage them to ask questions like:

- *Why do you do this at step 2?*
- *Why did you not consider starting with a different step?*
- *Was there a shorter way of getting to the answer?*

In Question A, since the same set of numbers is used, only plus and minus signs change; this is a good opportunity to observe if students are making sense of the operations.

- *What do the equations in Question A, parts (1) and (3) have in common?* (They are part of the same equation, $5x + 10 = y$. In the first equation, $y = 20$ and we are trying to find the corresponding value of x. In the second equation, $y = -20$ and we are trying to find the corresponding value for x.)
- *How else could we find the corresponding values for* x? (Note that this is the question posed in Question F. We could use a table or graph.)
- *What about the equations in Question A parts (2) and (4)? What do they have in common?*

In Question D, the Distributive Property is not needed for part (1). But the solution is much easier if it is applied to the left side of the equation first.

Caution: Students may have some confusion about applying the Distributive Property. We do not apply the Distributive Property to both sides of the equation. It is used to replace an expression with an equivalent expression—thus making the resulting equation easier to solve.

Be sure to discuss Question F:

Part (1) makes a connection back to using tables or graphs to find a solution. Students need to think of the equation, $5x + 10 = -20$ as a special case of $5x + 10 = y$. We are trying to find a value of x when the value of y is -20.

Part (2) raises the issue that the methods for solving linear equations work no matter what we call the variable—x, n, z, p, and so on.

Suggested Questions Put the following equation on the board: $3.2x + 5 = 16$.

- *How is this equation different from the equations in this problem?* (The coefficient of x is not a whole number.)
- *How can we solve this equation?* (The principles of equality work for any real number, so we can apply the principles of equality: $3.2x = 11$ $x = \frac{11}{3.2}$ or 3.4375.)

Put the following equation on the board and repeat the questions: $3.2x + 5 = 14.6$.

Repeat the questions for $3.2x + 15 = 6.5x + 10$. These questions will lead into the work in the next problem.

Check for Understanding

- *If* $y = 5x + -11$, *use a symbolic method to find* x *if* $y = 30$. *Use a symbolic method to find* x *if* $y = 23.5$.

Describe how you could use a graph or table to find the solutions.

If $y_1 = 5x + 11$ and $y_2 = 2x - (-18)$, use a symbolic method to find when $y_1 = y_2$.

3.4 Solving Linear Equations

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Develop strategies for solving linear equations
- Recognize subtle shifts in equations and how they affect the solution

Launch

Put up the first two equations from the Getting Ready:

$2x + 10 = 16$ $\quad\quad$ $2x - 10 = 16$

- *Compare these two equations. How would you solve these two equations?*

Compare the four equations:

$3x = 15$; $-3x = 15$; $3x = -15$; and $-3x = -15$

- *What does the 3x mean? What does the −3x mean? How would you solve each equation?*

Students can work in pairs and then move into larger groups of 4.

Materials

- Large sheets of poster paper
- Transparency 3.4

Vocabulary

- Properties of equality

Explore

Ask students to look at the four equations in Question A.

- *What do these equations have in common?*

Be sure students are making a record of their methods and checking their answers. Encourage students to look for commonalities and differences in each set of equations. If students are puzzled, ask:

- *How can you isolate the variable?*
- *Look at the equation. What is a good starting step?*
- *How can you get rid of or "undo" the operation, but still maintain equality?*

You could have different groups put up solutions to different parts of Questions A, B, C, D, and E on poster paper to refer to in the summary.

Summarize

Call on different groups to discuss their methods. Ask the class if they have any questions they want to ask the group. Questions like:

- *Why do you do this at step 2? Why did you not consider starting with a different step? Was there a shorter way of getting to the answer?*

In Question D part (1), the Distributive Property is not needed but the solution is easier if it is applied to the left side of the equation first. Discuss Question F: Students need to think of the equation, $5x + 10 = -20$, as a special case of $5x + 10 = y$. Part 2 raises the issue that the methods for solving linear equations work no matter what we call the variable—x, n, z, p, and so on. Put the following equation on the board: $3.2x + 5 = 16$.

- *How is this equation different from the equations in this problem? How can we solve this equation?*

These questions will lead into the work in the next problem.

Materials

- Student notebooks

ACE Assignment Guide for Problem 3.4

Differentiated Instruction Solutions for All Learners

Core 15–19, 21
Other *Applications* 20, 22, 23; *Connections* 34–36, 37; *Extensions* 44, 45; and unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 34–36: *Shapes and Designs*; 37: *Covering and Surrounding*

Answers to Problem 3.4

Checking is required, but not included in the answers.

A. 1. $x = 2$; $5x + 10 = 20$
$5x + 10 - 10 = 20 - 10$
$5x = 10$
$x = 2$

2. $x = 6$; $5x - 10 = 20$
$5x - 10 + 10 = 20 + 10$
$5x = 30$
$x = 6$

3. $x = -6$; $5x + 10 = -20$
$5x + 10 - 10 = -20 - 10$
$5x = -30$
$x = -6$

4. $x = -2$; $5x - 10 = -20$
$5x - 10 + 10 = -20 + 10$
$5x = -10$
$x = -2$

B. Note only one order of possible steps is given. Students may decide to solve the following equations using different steps.

1. $x = -2$; $10 - 5x = 20$
$-10 + 10 - 5x = 20 - 10$
$-5x = 10$
$x = -2$

2. $x = 6$; $10 - 5x = -20$
$-10 + 10 - 5x = -20 - 10$
$-5x = -30$
$x = 6$

C. 1. $x = 3$;
$4x + 9 = 7x$
$9 = 3x$
$3 = x$

2. $x = 2$
$4x + 9 = 7x + 3$
$4x + 6 = 7x$
$6 = 3x$
$2 = x$

3. $x = -3$
$4x - 9 = 7x$
$-9 = 3x$
$-3 = x$

4. $x = 2$
$4x - 9 = -7x + 13$
$4x - 22 = -7x$
$-22 = -11x$
$2 = x$

D. In part (1) students can divide both sides by 3 and proceed as in the previous problems. In parts (2) and (3), it is easier if they first apply the Distributive Property to the left side of the equation.

1. $x = 5$
$3(x + 2) = 21$ OR $3x + 6 = 21$
$x + 2 = 7$ $\quad 3x = 15$
$x = 5$ $\quad x = 5$

2. $x = 3$
$-3(x - 5) = 2x$
$-3x + 15 = 2x$
$-5x + 15 = 0$
$-5x = -15$
$x = 3$

3. $x = 7$
$5(x + 2) = 6x + 3$
$5x + 10 = 6x + 3$
$5x + 7 = 6x$
$7 = x$

E. 1. $x = 1.8$ or $\frac{9}{5}$ **2.** $x = 2.5$ or $\frac{10}{4} = \frac{5}{2}$

3. $x = -1.8$ or $\frac{-9}{5}$ **4.** $x = \frac{11}{12} \approx 0.917$

F. 1. To solve $5x + 10 = -20$, use the equation $5x + 10 = y$. To use a table, scan down the table of y values until you come to -20. The corresponding x value is the solution. To use a graph, graph the equation $5x + 10 = y$ until you have a point whose y-coordinate is -20. The corresponding x-coordinate is the solution.

2. It does not make a difference what letter or symbol is used to represent the variable; the principles of equality still apply.

Additional

Practice

Pages

Name ______________________________

Equation Tables

E 1-14
REASONING

1. Solve the equation $x - 3 = y$ for the given values of x or y to complete the table.

x	3	6	
y	0		6

2. Examine the completed table from Exercise 1. What would happen to the value of y if the value of x was increased by five?

____

____

3. How would the value of y change if the equation was rewritten as $x - y = 3$, the value of x was increased by five, and the equation was still true?

____

____

4. How would the value of y change if the equation was rewritten as $y + 3 = x$, the value of x was increased by five, and the equation was still true?

____

____

5. Compare the equations in Exercises 1–4. How are they alike? How are they different?

____

____

6. Solve the equation $n \div v = 2$ for the given values of n or v to complete the table.

n	4	10	
v	2		8

7. How would the value of v change if the value of n was doubled and the equation was still true?

____

____

Name ______________________________ Period ____________

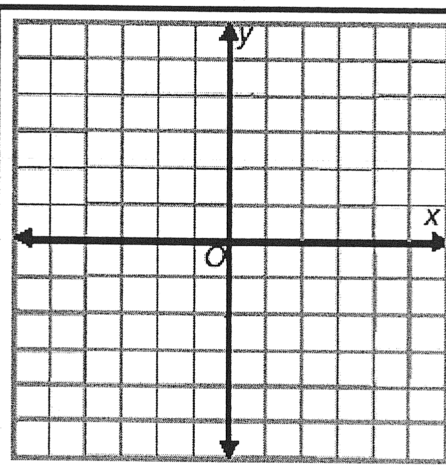
Review of Linear Representations

Given one of the representations below, find the other two.

Table | **Graph** | **Equation** $(y = mx + b)$

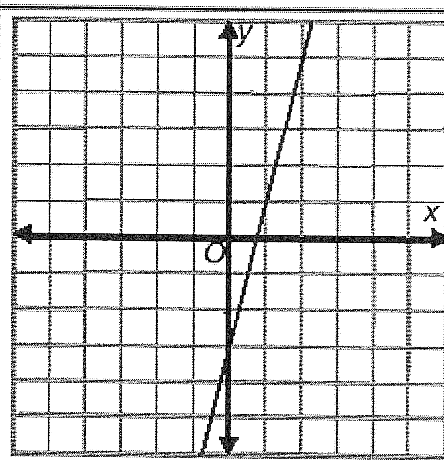
A.

x	y
−2	14
0	8
1	5
2	2
3	−1



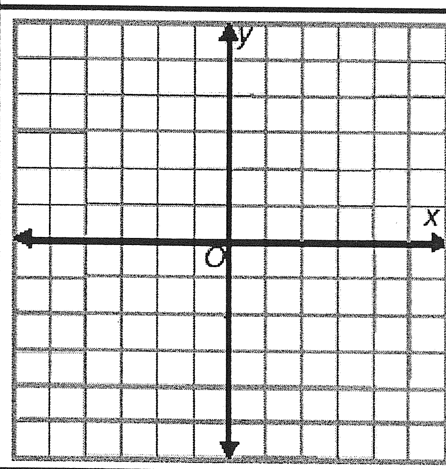
B.

x	y



C.

x	y



$y = \frac{1}{3}x + 1$

Which of the relationships shown above are proportional, *A, B,* and/or *C* ?

____

Name ______________________________ Period __________

Matching *Linear Representations*

Match a table (A–D) with a graph (E–H) and an equation (J–M). List your results below. For example, on the line for group 1 you should put 3 letters: one for a table, one for a graph, and one for an equation that all represent the same linear pattern.

	Table	Graph	Equation
Group 1:			
Group 2:			
Group 3:			
Group 4:			

A.

x	y
−2	−5
−1	−3
0	−1
1	1
2	3

B.

x	y
−2	3
−1	2
0	1
1	0
2	−1

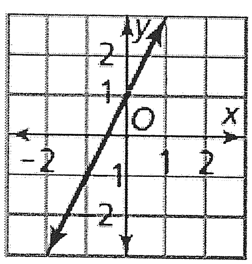
C.

x	y
−2	1.5
−1	1.5
0	1.5
1	1.5
2	1.5

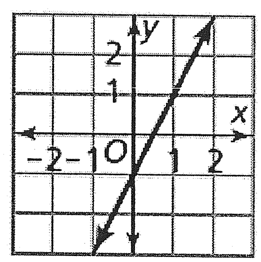
D.

x	y
−2	−3
−1	−1
0	1
1	3
2	5

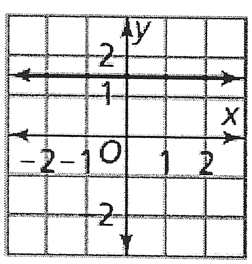
E.



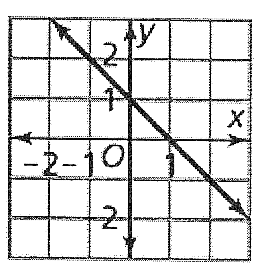
F.



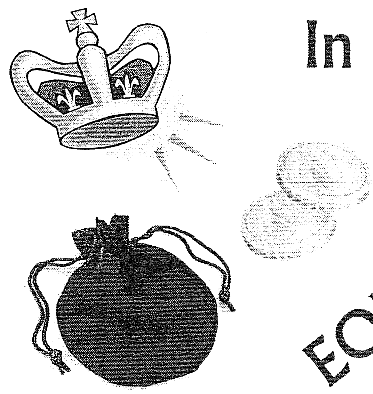
G.



H.



J. $y = 1.5$ **K.** $y = 2x - 1$ **L.** $y = 2x + 1$ **M.** $y = -x + 1$



In Search Of EQUALITY!

In the kingdom of Montarek, the ambassadors carry diplomatic pouches. The content of the pouches are unknown except by the ambassadors. Ambassador Milton wants to send one-dollar gold coins to another country.

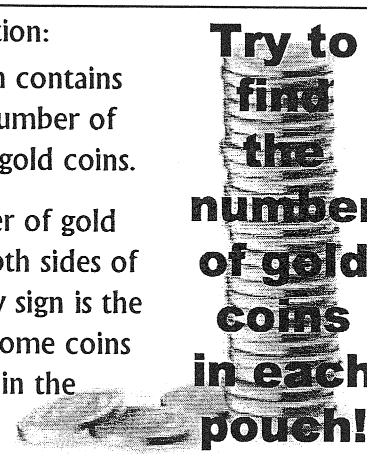
$1 gold coin

diplomatic pouch

His daughter, Sarah, is a mathematician. She helps him devise a plan based on *equality* to keep track of the number of one-dollar gold coins in each pouch.

In each situation:

- Each pouch contains the same number of one-dollar gold coins.
- The number of gold coins on both sides of the equality sign is the same, but some coins are hidden in the pouches.



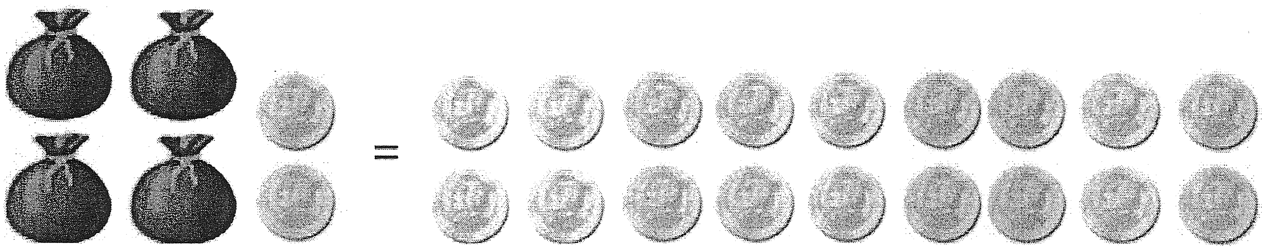
teacher information

Name ______________________________ Period ____________

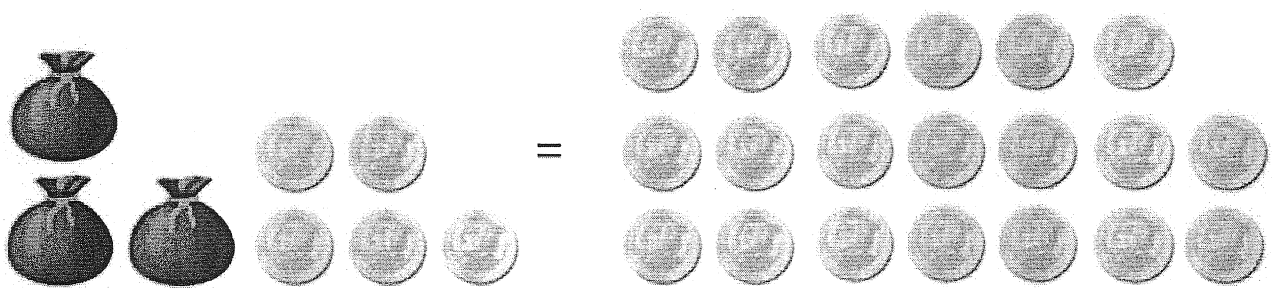
ACE #5-8 (CMP2) – Exploring Equality

For each situation in Exercises 5–8, find the number of coins in each pouch. Show your work underneath each set of pouches and coins.

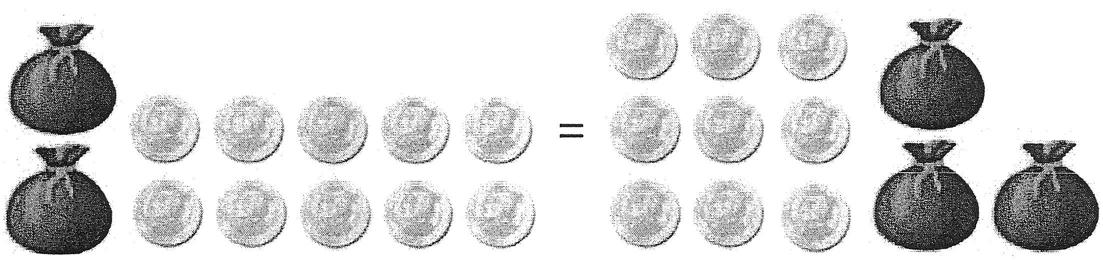
5.



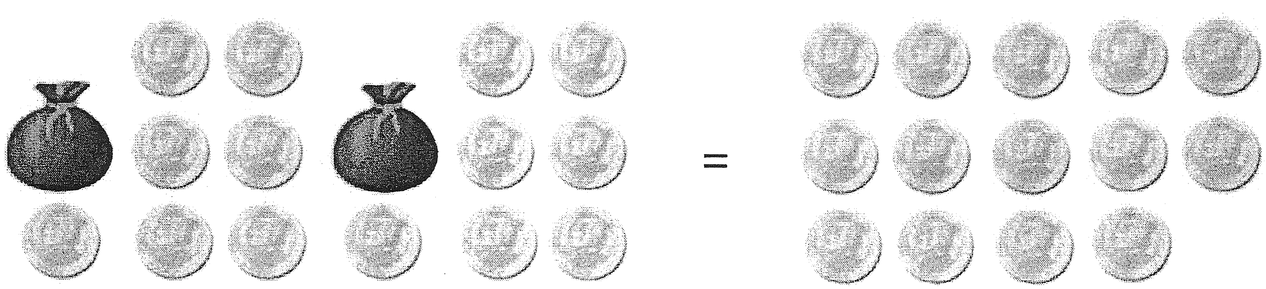
6.



7.



8.



Name ______________________________ Period __________

ACE #10 (CMP2) – More Exploring Equality

10. For each equation, sketch a picture using pouches and coins, and then determine how many coins are in a pouch.

a. $3x = 12$

b. $2x + 5 = 19$

c. $4x + 5 = 2x + 19$

d. $x + 12 = 2x + 6$

e. $3(x + 4) = 18$

Name ______________________________________ Period __________

ACE #11 (CMP2) – Mystery Numbers

11. For parts (a) and (b), find the mystery number by making an equation to match the clues...then solve.

a. If you add 15 to 3 times the mystery number, you get 78. What is the mystery number?

b. If you subtract 27 from 5 times the mystery number, you get 83. What is the mystery number?

c. Make up clues for a riddle whose mystery number is 9.

Name ____ Period ____________

ACE #12 (CMP2) – Solving for *X*

12. Solve each equation for x. Check your answers.

a. $7 + 3x = 5x + 13$

b. $3x - 7 = 5x + 13$

c. $7 - 3x = 5x + 13$

d. $3x + 7 = 5x - 13$

Name ______________________________ Period __________

ACE #14 (CMP2) – More Solving for *X*

14. Solve each equation for x. Check your answers.

a. $3x + 5 = 20$

b. $3x - 5 = 20$

c. $3x + 5 = -20$

d. $-3x + 5 = 20$

e. $-3x - 5 = -20$

Name ______________________________ Period _____

Even *More* Equality Practice!

Solve using the process of equality!

a) $3z - 19 = 173$

---

b) $4.5x = 45$

---

c) $67.1 = 29.7 - .2w$

---

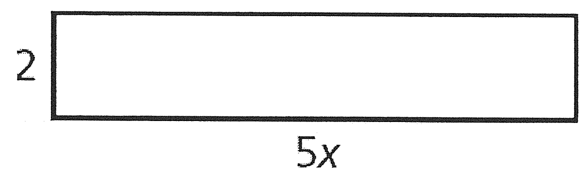
d) $2r + 10 = 22$

Name ______________________ Period __________

ACE #37 (CMP2) – Solving for *X* With Shapes

37. The perimeter of each shape is 24 cm. Find the value of x.

a.



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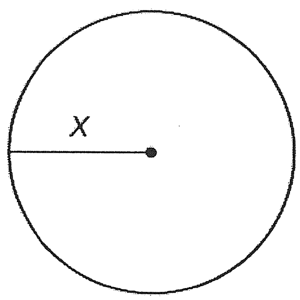
b.



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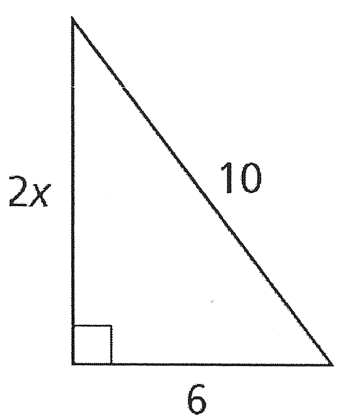
c.



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d. Find the area of each figure in parts (a)–(c).

a. ______________ b. ______________ c. ______________

Filling & Wrapping

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations /Additional Targets
CMP2 Additional Practice Inv 2- to review surface area and volume of rectangular prisms from Inv 1,2,3 completed in 6th grade (may want to skip question 1 or make optional)	1	Binder/CMP2 disc (7)		
Topic 6: Investigating Volume ***May want to do after 4.1***	1	Online lesson		**7.3.A Determine the surface area and volume (including appropriate units) of a cylinder using the appropriate formulas and explain why the formulas work.**
Problem 4.1, Filling a Cylinder, p. 38	1		Should Do #1	
Problem 4.2, Making a Cylinder from a Flat Pattern, p. 39	1		Must Do	
Problem 4.3, Designing a New Juice Container, p. 40	1		Should Do	**7.3.B Determine the volume of pyramids and cones using formulas.**
Investigation 4 Mathematical Reflections, p. 45	1			
Problem 5.1, Comparing Spheres & Cylinders, p. 47	1		Must Do	**7.3.C Describe the effect that a change in scale factor on one attribute of a two- or –three dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.**
Problem 5.2, Comparing Cones & Cylinders, p. 49	1		Should Do	
Problem 5.3, Melting Ice Cream, p. 50	1		Must Do	
Investigation 5 Mathematical Reflections, p. 56	1			
Topic 6: Investigating Volume	**1**	Online lesson		
Volume of Triangular prisms Additional Practice CMP2 p127 and "volume of Triangular Prisms" worksheet p91	1	CMP2 disc(7) Notebook		**7.3.D Solve single- and multi- step word problems involving surface area or volume and verify the solutions.**
Review Surface Area of Pyramids Worksheet	1	Notebook		
VOLUME of a Pyramid –worksheet p92 (and mixed practice wksht p95)	1	Notebook		Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Problem 6.1, Building a Bigger Box, p. 58	1		Must Do	
Problem 6.2, Scaling Up the Compost Box, p. 59	1		Should Do	
Problem 6.3, Looking at Similar Prisms, p. 60	1		Must Do	
Teach HOW TO USE formula sheet provided with MSP ; also additional practice worksheets included at end of section	1	MSP		
Unit Assessment	**2**			
Review & Reflect Assessment, Student Self-Assessment	1			
Total Instructional Days for Filling & Wrapping:	**21**			

Contents in Filling and Wrapping

Name ______________________ Date __________ Class __________

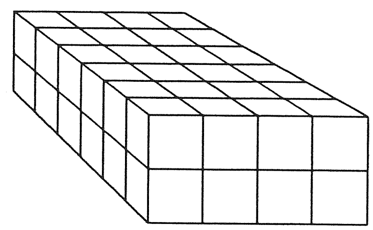
Additional Practice

Investigation 2

Filling and Wrapping

1. The bottom of a closed rectangular box has an area of 50 square centimeters. If the box is 8 centimeters high, give at least three possibilities for the dimensions of the box.

2. a. The rectangular prism at the right is made from centimeter cubes. What are the dimensions of the prism?

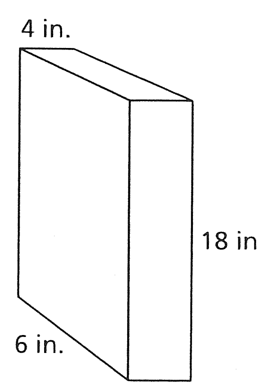


   b. What is the surface area of the prism?

   c. What is the volume of the prism? That is, how many cubes are in the prism?

   d. Give the dimensions of a different rectangular prism that can be made from the same number of cubes. What is the surface area of the prism?

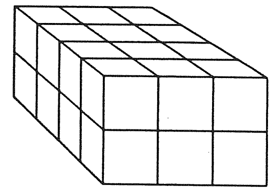
3. Use the diagram at the right to answer the following questions.



   a. What is the total surface area of the box, including the bottom and the top?

   b. How many inch cubes would it take to fill the box? Explain your reasoning.

4. a. Each small cube in the rectangular prism at the right has edges of length 2 centimeters. What are the dimensions of the prism in centimeters?

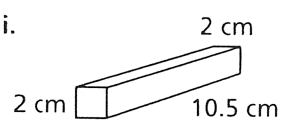


b. What is the surface area of the prism in square centimeters?

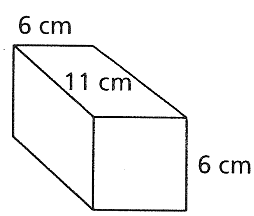
c. How many 1-centimeter cubes would it take to make a prism with the same dimensions as this prism? Explain your reasoning.

5. Answer parts (a) and (b) for each closed box below.

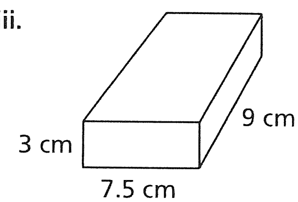
i.



ii.



iii.



a. What is the surface area of each box?

b. What is the volume of each box

Topic 6: Investigating Volume

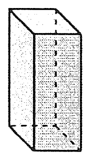
for use before ***Looking for Pythagoras*** **Investigation 3**

Volume is the amount of space enclosed in a solid. It is expressed in cubic units. The volume of the Empire State Building in New York City is about 37 million cubic feet. The volume of a raisin is about $\frac{1}{8}$ cubic inch.

Problem 6.1

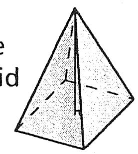
a. The volume of a prism or a pyramid with a square base is determined by the height of the solid and the area of the base.

Rectancle prism



Volume = Base × height
Volume = length × width × height
= lwh

Square Pyramid



Volume = $\frac{1}{3}$ × Base × height
= $\frac{1}{3}s^2h$

A. What is the volume of a rectangular prism with a length of 8 cm, a width of 4 cm, and a height of 10 cm?

B. What is the volume of a square pyramid with a height of 3 feet and a base with sides of 1 foot?

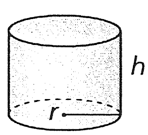
C. Give two sets of possible dimensions for a rectangular prism with a volume of 100 in^3.

D. How does the volume of a rectangular prism change if you lay it on its side before taking measurements? Explain.

Problem 6.2

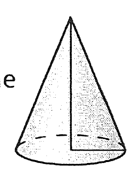
The volume of a cone or a cylinder is determined by the height and the area of the base. The exact volume of a cone or cylinder includes the value π.

Cylinder



Volume = Base × height
= $\pi r^2 h$

Cone



Volume = $\frac{1}{3}$ × Base × height
= $\frac{1}{3}\pi r^2 h$

A. 1. What is the exact volume of a cylinder with radius 5 m and height 3 m?

2. What is the exact volume of a cone with radius 7 in. and height 12 in.?

B. If a cone and a cylinder have the same radius and the same height, how many times greater is the volume of the cylinder than the cone?

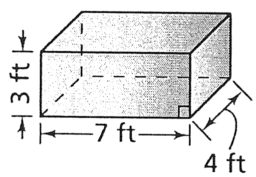
C. If the volume and height of a cone and a cylinder are the same, which one has the larger base?

D. If the radius of a cone doubles and the height remains the same, what happens to the volume?

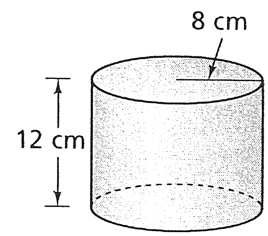
Exercises

Find the exact volume of each solid.

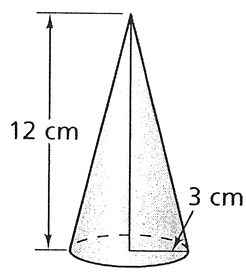
1.



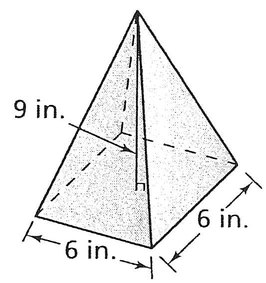
2.



3.



4.



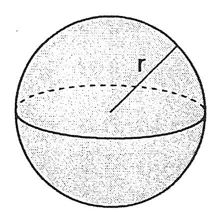
5. How are the formulas for the volume of prisms and cylinders the same?

6. How are the formulas for the volume of cones and pyramids the same?

7. A watering can does not fit under a faucet, so Trang is using a paper cone to fill it with water. The cone has a radius of 1 inch and a height of 4 inches. The can has a radius of 2 inches and a height of 8 inches. How many conefuls of water will it take to fill the can?

8. The Great Pyramid of Khufu in Egypt has a square base with sides of about 230 meters and a height of about 146 meters. What is the approximate volume of the pyramid?

9. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.



a. What is the volume of a baseball with a diameter of 2.8 inches?

b. About many times more volume does a basketball with a radius of 4.78 inches have than a baseball?

10. What is the exact volume of the largest cone that can fit into a cube with sides of 10 inches?

Topic 6: Investigating Volume

At a Glance

PACING 1 day

Mathematical Goals

- Determine the volume of three-dimensional shapes.
- Investigate the relationship of the volume of three-dimensional figures with the same base area and height.

Guided Instruction

Use unit cubes in an exercise to give the students a hands-on experience with cubic measure. Show that the cubes can be rearranged into numerous configurations without the volume changing.

The solid shapes used in this lesson are all right shapes: the bases of the rectangular prisms and cylinders are aligned vertically; the apexes of the pyramids and cones are directly above the center of the base. Even if these shapes leaned to one side, the same formulas for volume would apply.

After Problem 6.1

- *Why doesn't it matter which edges you call length, width, and height when you find the volume of a rectangular prism?* (According to the Commutative Property of Multiplication it doesn't matter in what order you do the multiplication.)
- *How do you define a rectangular prism?* (a solid figure with six faces that are rectangles)
- *What do you call a rectangular prism whose sides are all squares?* (cube)
- *What is the definition of a cubic inch?* (The volume of a cube with sides of 1 inch × 1 inch.)

You will find additional work on volume in the grade 7 unit *Filling and Wrapping.*

Vocabulary

- volume

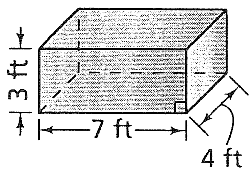
Materials

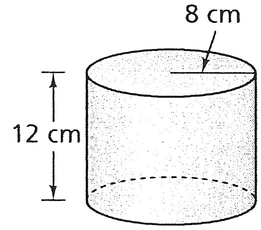
- Labsheet 6ACE Exercises 1–4

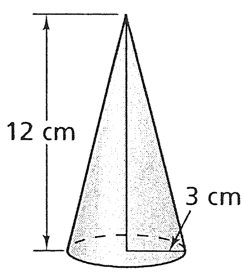
Name ______________________ Date ____________ Class ____________

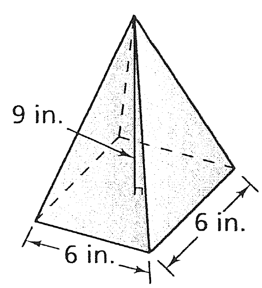
Labsheet 6ACE Exercises 1–4

Topic 6

1. 

2. 

3. 

4. 

Assignment Guide for Topic 6

Core 1–9
Advanced 10

Answers to Topic 6

Problem 6.1

A. 1. 320 cm^3

B. 1 ft^3

C. Answers may vary. Sample: 5 in. × 5 in. × 4 in.; 10 in. × 5 in. × 2 in.

D. The volume does not change if you place a rectangular prism on its side before measuring.

Problem 6.2

A. 1. 75π m^3

2. 196π $in.^3$

B. 3 times

C. the cone

D. Volume increases by a factor of 4.

Exercises

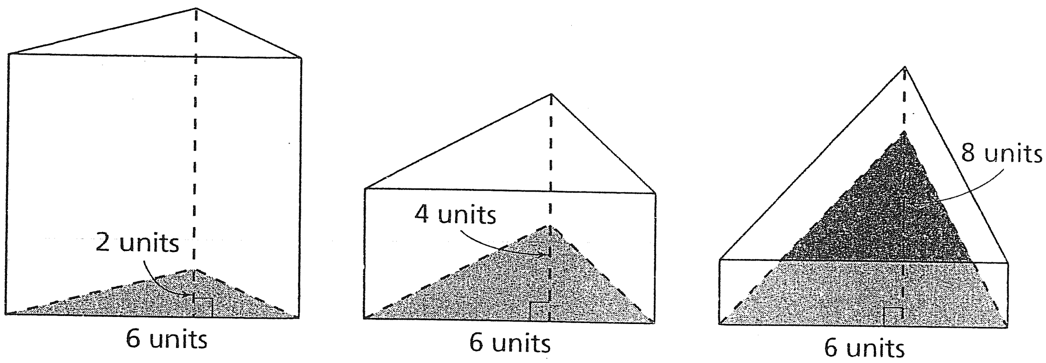
1. 84 ft^3
2. 768π cm^3
3. 36π cm^3
4. 108 $in.^3$
5. They both multiply the area of a base times the height.
6. They both multiply $\frac{1}{3}$ the area of a base times the height.
7. 24 conefuls
8. 2,574,467 m^3
9. **a.** 11.5 $in.^3$

   b. 40 times more volume
10. $\frac{250\pi}{3}$ $in.^3$

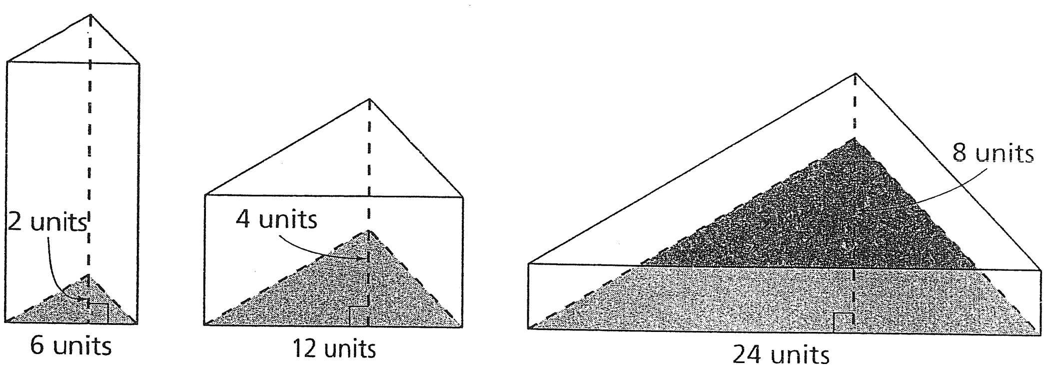
Name ______________________ Date ____________ Class ____________

Additional Practice *(continued)*

8. Below are three triangular prisms (not drawn to scale). The height of the first prism is 8 units, and the volumes of all three prisms are the same. What are the heights of the other two prisms?



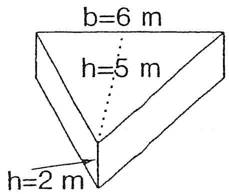
9. Below are three triangular prisms (not drawn to scale). The height of the first prism is 8, and the volumes of all three prisms are the same. What are the heights of the other two prisms?



Name ______________________________

Volume of a Triangular Prism

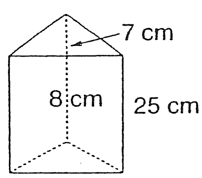
To find the volume of a triangular prism, use the formula **Volume = B • h**, where B is the area of the base of the prism and h is the height of the prism. The base of a triangular prism is a triangle.



Step #1 Area of the Base	**Step #2 Volume**
Base = $\frac{1}{2}$ • base of triangle • height of triangle	Volume = Base • height
Base = $\frac{1}{2}$ • 6 m • 5 m	Volume = 15 m^2 • 2 m
Base = $\frac{1}{2}$ • 30 m^2	Volume = 30 m^3
Base = 15 m^2	

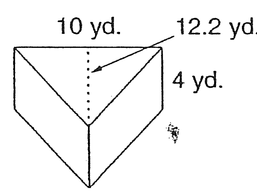
Find the volume.

A.



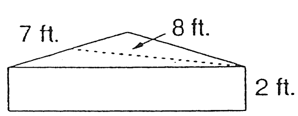
V = ____________

B.



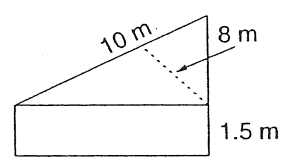
V = ____________

C.



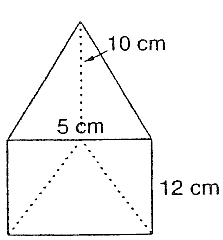
V = ____________

D.



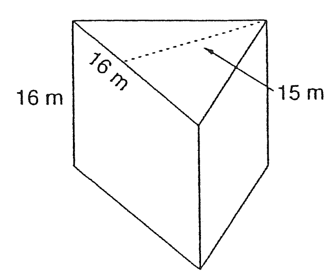
V = ____________

E.



V = ____________

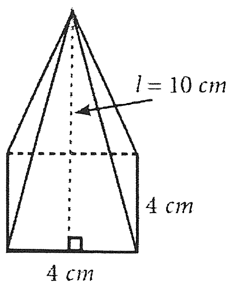
F.



V = ____________

Name ____ Period ______________

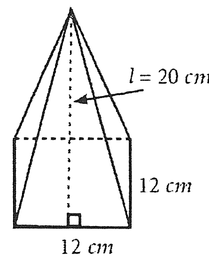
Review: Surface Area of Pyramids



a. Draw a net of the pyramid.

b. Find the area of each figure in the net.

c. Find the surface area of the pyramid.

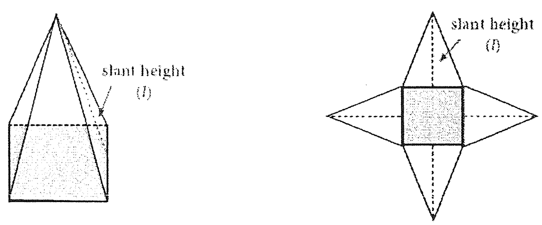


a. Draw a net of the pyramid.

b. Find the area of each figure in the net.

c . Find the surface area of the pyramid.

The slant height of a pyramid is the height of a lateral face. The variable *l* is used to represent slant height. The net of a square pyramid is shown below.



Sam made game pieces in the shape of square pyramids. Each piece has a base edge of 3 cm and a slant height of 4 cm. He will paint all of the pieces. He needs to know how much paint he needs.

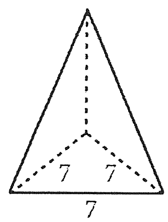
a. Find the surface area of one game piece.

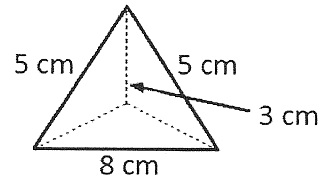
b. Each game has 24 game pieces. Find the total surface area of one set of game pieces.

c. He wants to make 12 games. What is the total surface area for all 12 games?

d. A can of paint covers 400 square centimeters. How many cans of paint will he need?

Slant height = 11





a. **Draw a net of the pyramid.**

b. **Find the area of each figure in the net.**

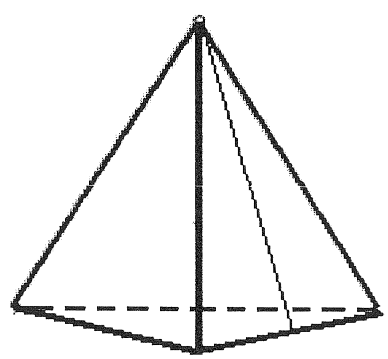
c. **Find the surface area of the pyramid.**

a. **Draw a net of the pyramid.**

b. **Find the area of each figure in the net.**

c. **Find the surface area of the pyramid.**

Tanya wants to wrap the box shaped like a triangular pyramid below. Each triangular side has a height of 7 in., with the sides of the base each being 10 in. How much surface area will she need to cover?



Name ____

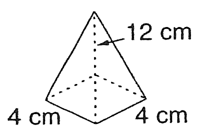
Volume of a Pyramid

To find the volume of a pyramid, use the formula **Volume = $\frac{1}{3}$ • Base • height,** where B is the area of the base and h is the height of the pyramid.

Step #1 Find the Area of the Base	**Step #2 Find the Volume of the Pyramid**
If the base is a rectangle, use the formula **B = length • width.** If the base is a triangle, use the formula **B = $\frac{1}{2}$ • b • h.**	**V = $\frac{1}{3}$ • Base • height of the Pyramid**

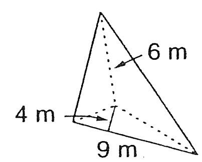
Find the volume of each pyramid. Use the formula **V = $\frac{1}{3}$ • Base • height**

A.



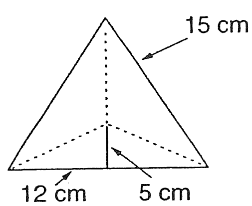
V = ______________________

B.



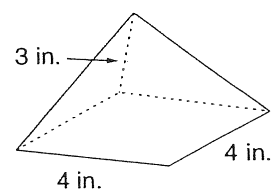
V = ______________________

C.



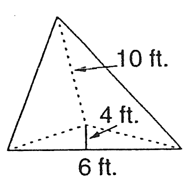
V = ______________________

D.



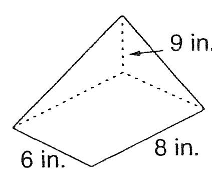
V = ______________________

E.



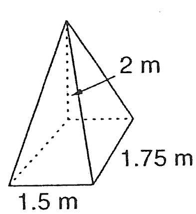
V = ______________________

F.



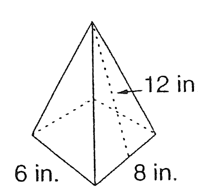
V = ______________________

G.



V = ______________________

H.



V = ______________________

Name ____

Volume: Mixed Practice

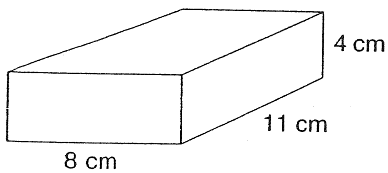
Find the volume of each figure. You may use a calculator. Round your answers to the nearest hundredth.

The volume of a prism or cylinder is **Base • height.**

The volume of a pyramid or cone is $\frac{1}{3}$ **• Base • height.**

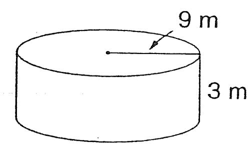
Remember! **Base** means the area of the base.

A.



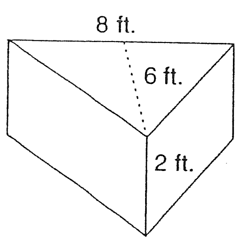
V = ____________________

B.



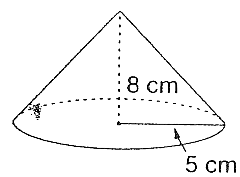
V = ____________________

C.



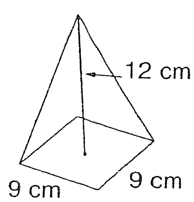
V = ____________________

D.



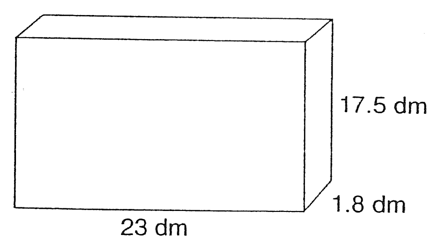
V = ____________________

E.



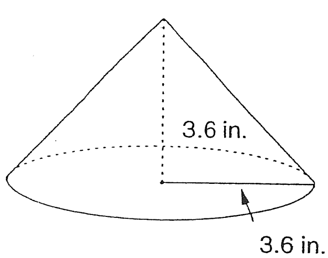
V = ____________________

F.



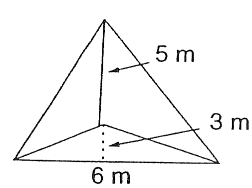
V = ____________________

G.



V = ____________________

H.



V = ____________________

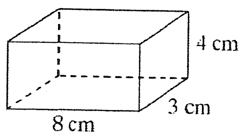
Additional Practice Pages

Name ______________________________ Date __________ Class __________

Review: Surface Area and Volume of Rectangular Prisms

Find the surface area and volume of each rectangular prism.

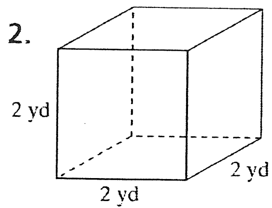
1.



Surface Area ______________

Volume ______________

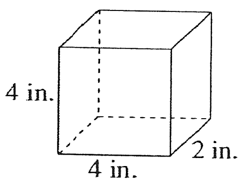
2.



Surface Area ______________

Volume ______________

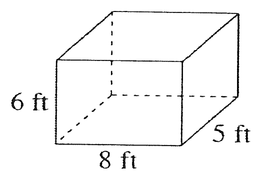
3.



Surface Area ______________

Volume ______________

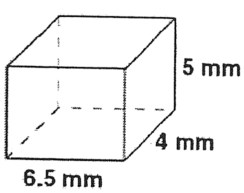
4.



Surface Area ______________

Volume ______________

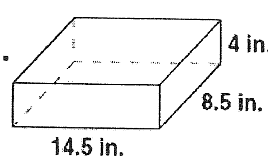
5.



Surface Area ______________

Volume ______________

6.



Surface Area ______________

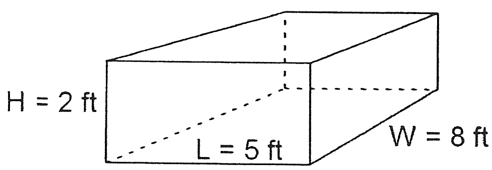
Volume ______________

Name ______________________________ Period __________

Filling and Wrapping – Volume of Solids

d the <u>**VOLUME**</u> of each of the following solids.

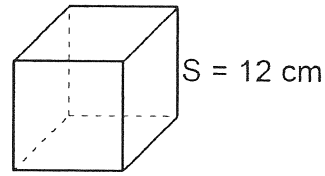
Rectangular Prism



1) FORMULA:

VOLUME =

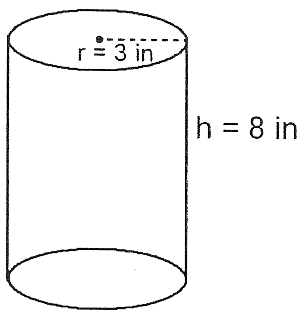
Cube



2) FORMULA:

VOLUME –

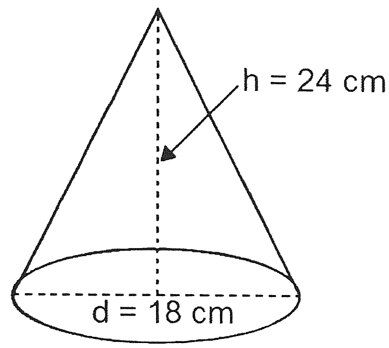
Cylinder



3) FORMULA:

VOLUME –

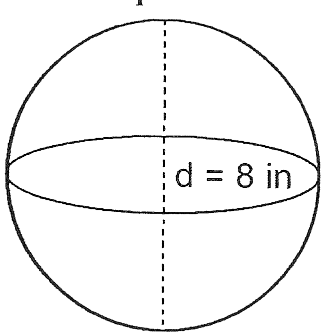
Cone



4) FORMULA:

VOLUME –

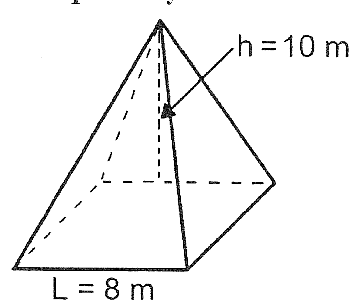
Sphere



5) FORMULA:

VOLUME =

Square Pyramid



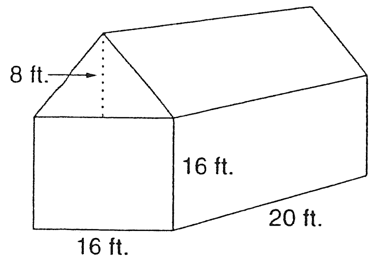
6) FORMULA:

VOLUME =

Total Volume

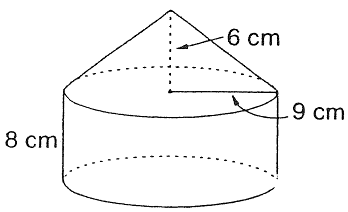
To find the total volume for the figures pictured, find the volume of each solid figure that makes up the figure. Then add the volumes. You may use a calculator. Record each formula that you use and the total volume in the space beside the diagram.

A.



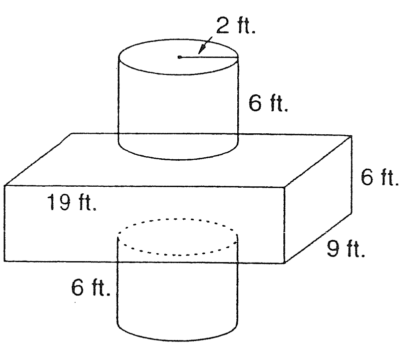
V = ______________

B.



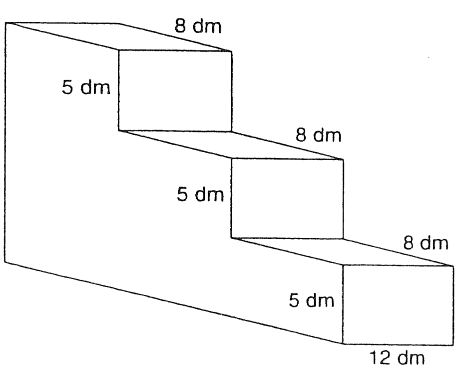
V = ______________

C.



V = ______________

D.



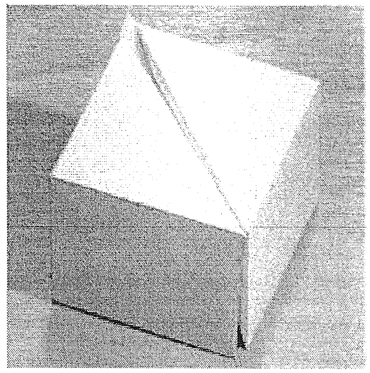
V = ______________

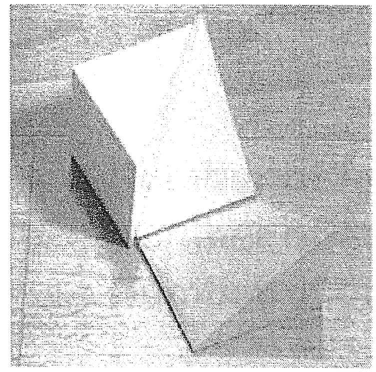
Paper Model of a Three Pyramids That Form A Cube

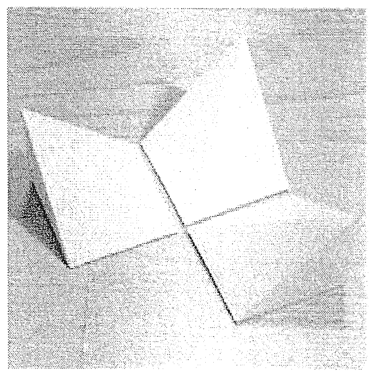
http://www.korthalsaltes.com/model.php?name_en=three%20pyramids%20that%20form%20a%20cube

Three Pyramids That Form A Cube:

Number of faces: 5
Number of edges: 8
Number of vertices: 5







Volume of a pyramid: V = 1/3 B h

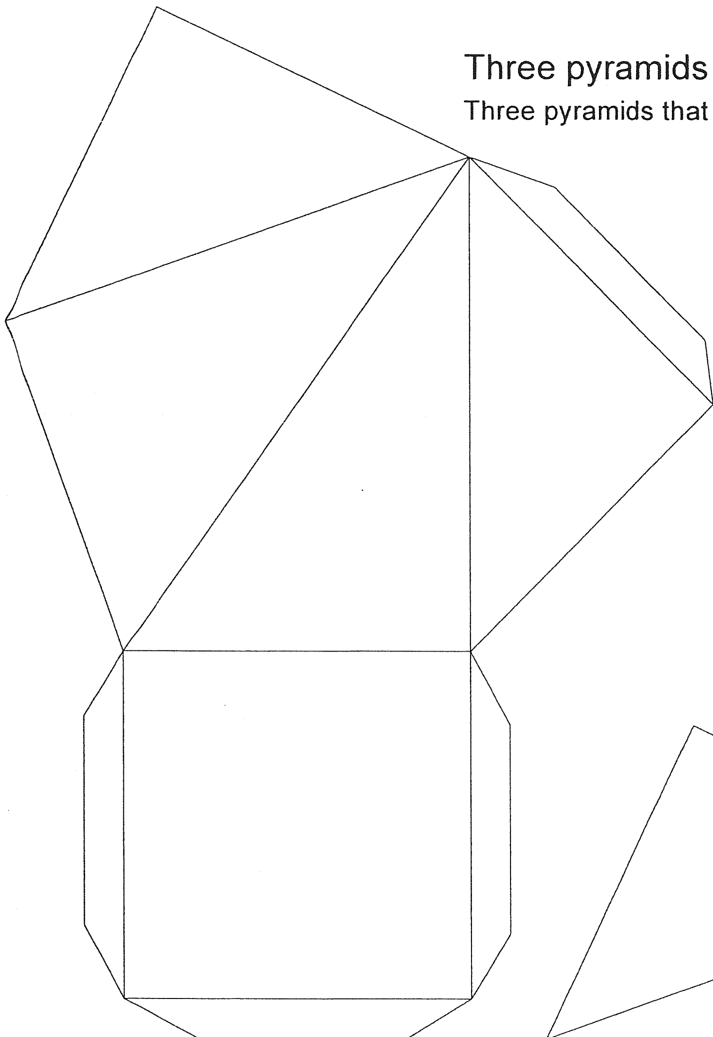
V - Volume
B - The surface area of the base
h - Height of the pyramid

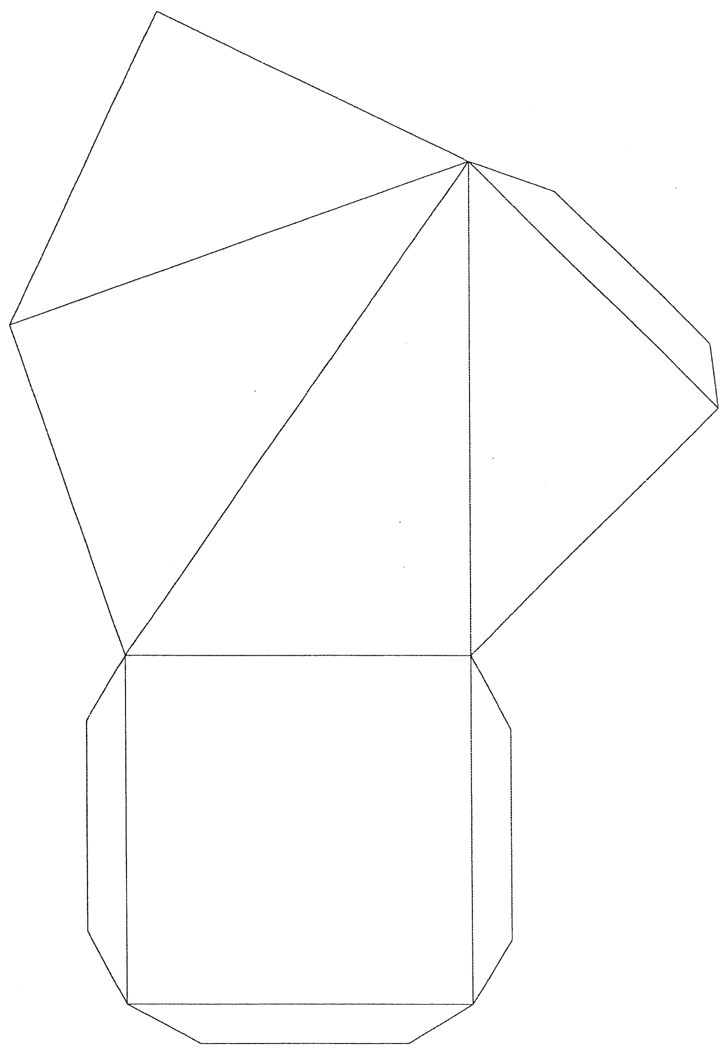
Definition of a Pyramid:

A pyramid is a polyhedron with one face (known as the "base") a polygon and all the other faces triangles meeting at a common polygon vertex (known as the "apex").

Three pyramids

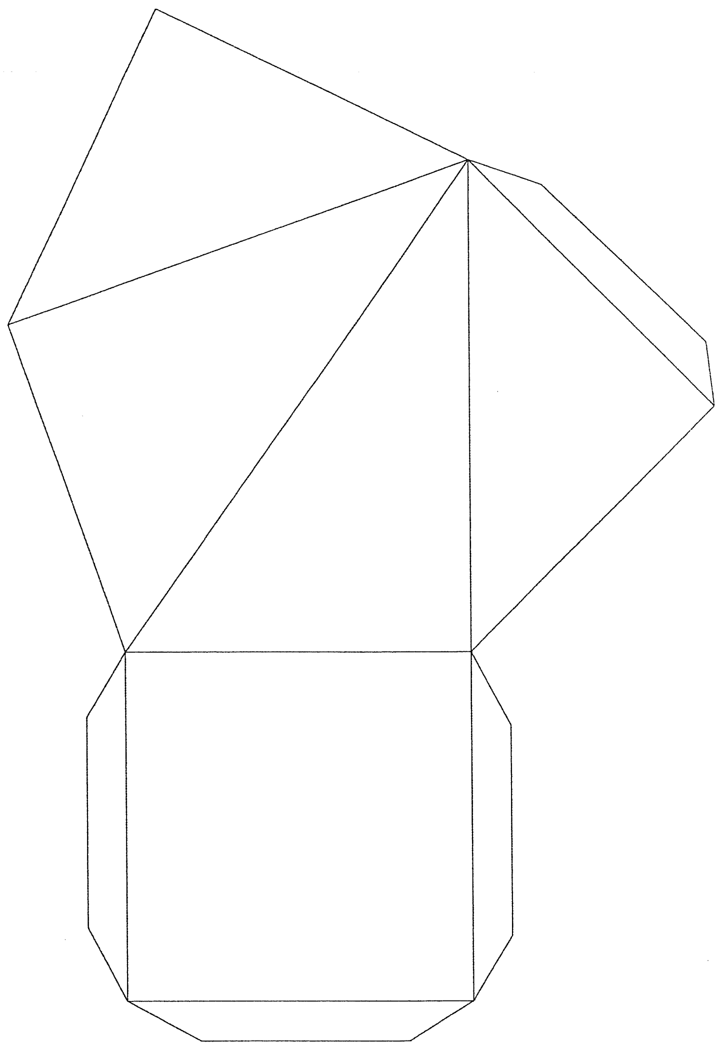
Three pyramids that fit in one cube





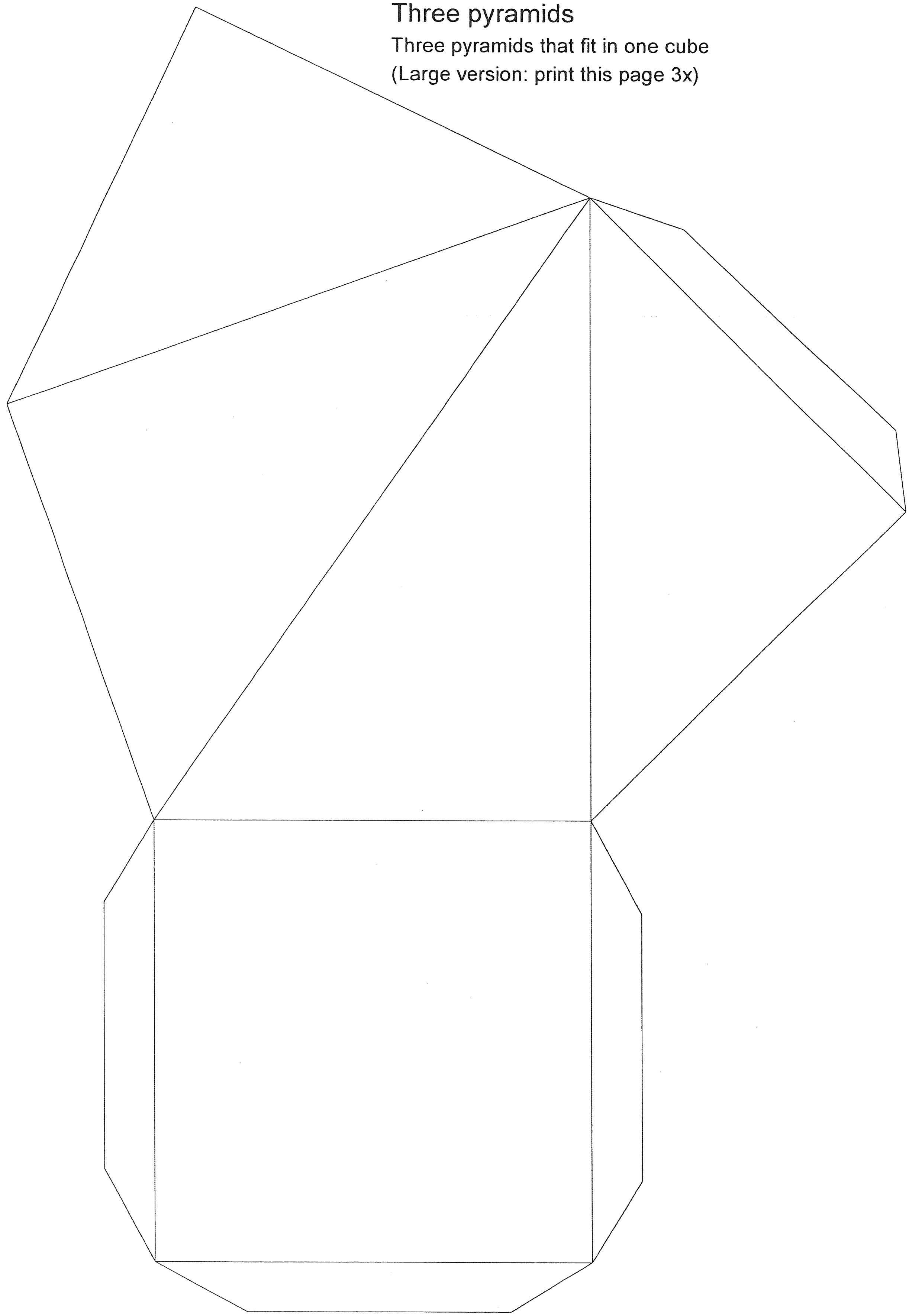
Three pyramids

Three pyramids that fit in one cube



Three pyramids

Three pyramids that fit in one cube
(Large version: print this page 3x)



Data Distributions CMP2 (also includes lessons from Data About Us, and Samples & Populations)

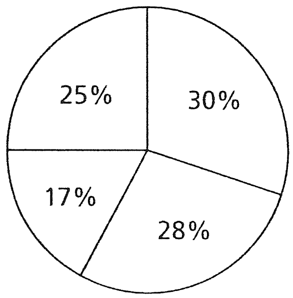
Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations /Additional Targets
Topic 15: Circle Graphs	**2**	Online lesson		**7.4.C Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range) and evaluate the suitability and limitations of using each measure for different situations.** **7.4.D Construct and interpret histograms, stem-and-leaf plots, and circle graphs.** **7.4.E Evaluate different displays of the same data for effectiveness and bias, and explain reasoning.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Problem 1.1, Variability in Categorical Data, p.6	2			
Problem 1.2, Variability in Numerical Counts, p.8	1			
Problem 1.3, Variability in Numerical Measurements, p.12	1			
Problem 1.4, Two Kinds of Variability, p.15	1			
Mathematical Reflection, p.27	½			
Problems 2.1, The Mean as an Equal Share, p.29	1			
Problem 2.2, The Mean as a Balance Point in a Distribution, p,.32	1			
Problem 2.3, Repeated Values in a Distribution, p.36	2			
Problem 2.4, Measures of Center and Shapes of Distributions, p.40	2			
Mathematical Reflections, p.54	½			
Check-Up 1	½	Binder/ teacher express		
Data About Us—CMP2 Problem 2.1 Traveling to School /Making a Stem and Leaf Plot, p. 30-33	1	Binder/CMP2 Disc(6)		
Data About Us—CMP2 Problem 2.2 Jumping Rope/Comparing Distributions, p. 34-35	1	Binder/CMP2 Disc(6)		
Samples & Populations—CMP2 Problem 1.1, From Line Plots to Histograms, p. 8	2	Binder/CMP2 Disc(8)		
Samples & Populations—CMP2 Problem 1.2, Using Histograms, p. 10	1	CMP2 Disc(8)		
Topic 10: Misleading Data Displays	1	Online lesson		
Topic 6: Misleading Graphs	1	Online lesson		
Additional Practice pages		binder		
Review & Reflect Assessment, Student Self-Assessment	1			
Data Distributions Unit Assessment	2			
Total Instructional Days for Data Distributions	**25**			

Contents in Data Distributions

Topic 15: Circle Graphs

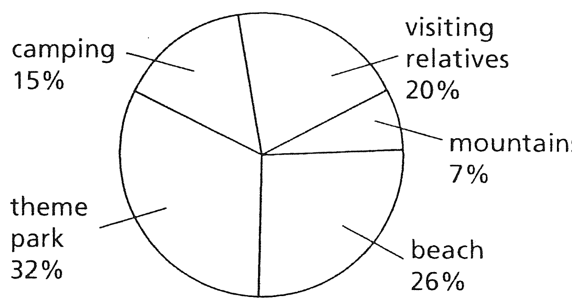
for use before ***Data Distributions*** **Investigation 1**

Circle graphs, or pie charts, show data by the percent of a quantity divided into several categories. A circle graph always represents all (100%) of the data.



Problem 15.1

A. The circle graph shows the vacation choices for 200 middle school students.



1. What percent of the students like to go to a theme park on a vacation?
2. Of the 200 students, how many students preferred to go to the beach on their vacation?
3. Of the 200 students, how many students like to go camping or to the mountains? Explain.

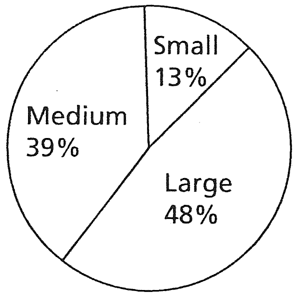
B. Nate goes to Westville Middle School. He wants to make a circle graph to represent the three grades of his school.

Grade	Percent of students
6	33%
7	42%
8	25%

1. Nate knows that 50%, or one half, of a circle is a straight line. How many degrees are there in a straight line?
2. Multiply the percent of students in the eighth grade by 360 to find the central angle for that piece of the circle.
3. Find the central angle for each of the other two grades.
4. Use a compass and protractor to draw a circle graph.
5. Label each sector with the appropriate grade and percent.

Exercises

1. The circle graph shows the size of the businesses in Westville that have made a decision to "Go Green."



a. What percent of the businesses that decided to "Go Green" were small?

b. There are 170 businesses that participated. How many medium-sized businesses decided to "Go Green?"

2. a. Make a circle graph for the table of data.

Homeroom Teacher	Number of students
Steel	20
Anderson	38
Payne	28
Johnson	36
Harmon	42
Martin	36

b. Make a circle graph for the table of data.

Favorite color	Percent of students
green	26%
purple	13%
pink	12%
blue	30%
red	19%

c. Determine the actual number for each color group.

4. Explain how a circle graph represents data differently from a bar or line graph.

Topic 15: Circle Graphs

At a Glance

PACING 1 day

Mathematical Goals

- Read and organize data in circle graphs.

Guided Instruction

Vocabulary
- circle graph

Materials
- compass
- protractor

Because circle graphs depend on fractions, percents, and central angles of a circle, you should review the connections between these topics.

Start with a clock as a point of reference.

- *A clock is a circle divided into segments by numbers. How many segments are on a clock face?* (12)
- *What fractional part of the entire clock face is each segment?* $\left(\frac{1}{12}\right)$
- *What fractional part of the circle is between 12 and 3?* $\left(\frac{3}{12}, \text{ or } \frac{1}{4}\right)$
- *How do you write* $\frac{1}{4}$ *as a percent?* (25%)
- *If one clock hand is on the 12 and the other is on the 3, what is the name of the angle that is formed in the center of the clock face?* (right angle)
- *How many degrees are in a right angle?* (90°)

Continue relating $\frac{1}{4}$ to 25% to 90° to establish the understanding of representing percents within a circle.

If it has been a while since the students have used a compass and/or a protractor, you may need to give them an opportunity to practice drawing circles and central angles.

Let the students work in pairs. It would be a good idea to check each student's work at Question B, part 2. If any student has an incorrect answer to this question, they will need assistance finding the central angles.

Summarize with questions like:

- *When would you use a circle graph to display data?* (When you want to see how each part compares to the whole.)
- *How is the data expressed?* (percents)
- *What do you need to find before you can display the data in the circle graph?* (the central angle)
- *How can you find a central angle?* (by multiplying the percent by 360°)

You will find additional work on circle graphs in the grade 6 unit *Data About Us.*

ACE Assignment Guide for Topic 15

Core 1–3

Answers to Topic 15

Problem 15.1

A. 1. 32%

2. 52

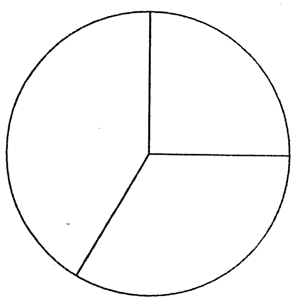
3. 44 students; Explanations may vary. Sample: I added the 7% of the Mountains to the 15% of Camping for a total of 22%. 22% of 200 students is 44 students.

B. 1. 180°

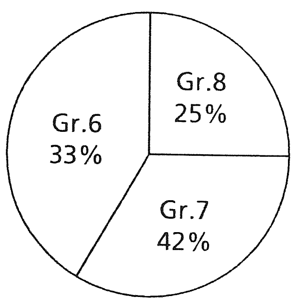
2. $0.25 \times 360° = 90°$

3. $0.33 \times 360° = 120°$; $0.42 \times 360° = 150°$

4.



5.

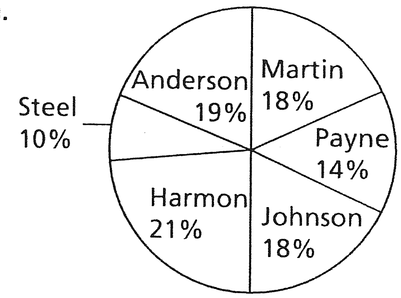


Exercises

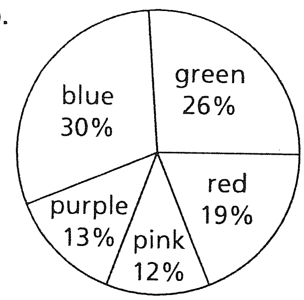
1. a. 13%

b. 66.3, or 66 businesses

2. a.



b.



c. student total 200; green 52, purple 26, pink 24, blue 60, and red 38.

3. Answers may vary. Sample: A circle graph compares parts of a whole and visually shows relationships of one category to another. A bar graph is good for ordering the data, but does not give a good visual of the whole. A line graph usually shows changes over a progression of time for the left to the right.

Investigation 2

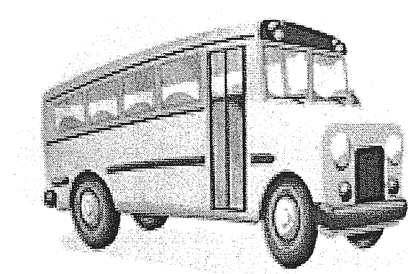
Using Graphs to Explore Data

Sometimes data may be spread out. When these data are displayed on a line plot or a bar graph, it is not easy to see patterns. In this investigation, you will learn how to highlight data using displays called stem-and-leaf plots and back-to-back stem-and-leaf plots to help you see patterns.

In Investigation 1, you analyzed single sets of data. Sometimes you may want to analyze whether there is a relationship between two different data sets. In this investigation, you will learn how to display data pairs from two different data sets using a coordinate graph.

2.1 Traveling to School

While investigating the times they got up in the morning, a middle-school class was surprised to find that two students got up almost an hour earlier than their classmates. These students said they got up early because it took them a long time to get to school. The class then wondered how much time it took each student to travel to school. The data they collected are on the next page.



Getting Ready for Problem 2.1

Use the table on the next page to answer these questions:

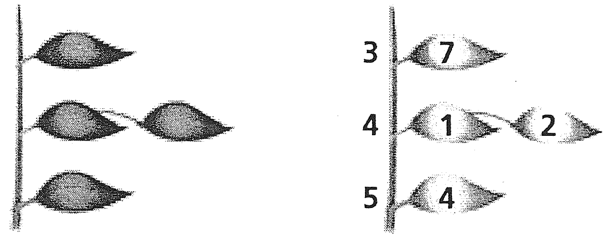
- What three questions did the students ask?
- How might the students have collected the travel-time data?
- Would a line plot be a good way to show the data? Why or why not?

Times and Distances to School

Student's Initials	Time (minutes)	Distance (miles)	Mode of Travel
DB	60	4.50	Bus
DD	15	2.00	Bus
CC	30	2.00	Bus
FH	35	2.50	Bus
SE	15	0.75	Car
AE	15	1.00	Bus
CL	15	1.00	Bus
LM	22	2.00	Bus
QN	25	1.50	Bus
MP	20	1.50	Bus
AP	25	1.25	Bus
AP	19	2.25	Bus
HCP	15	1.50	Bus
KR	8	0.25	Walking
NS	8	1.25	Car
LS	5	0.50	Bus
AT	20	2.75	Bus
JW	15	1.50	Bus
DW	17	2.50	Bus
SW	15	2.00	Car
NW	10	0.50	Walking
JW	20	0.50	Walking
CW	15	2.25	Bus
BA	30	3.00	Bus
JB	20	2.50	Bus
AB	50	4.00	Bus
BB	30	4.75	Bus
MB	20	2.00	Bus
RC	10	1.25	Bus
CD	5	0.25	Walking
ME	5	0.50	Bus
CF	20	1.75	Bus
KG	15	1.75	Bus
TH	11	1.50	Bus
EL	6	1.00	Car
KLD	35	0.75	Bus
MN	17	4.50	Bus
JO	10	3.00	Car
RP	21	1.50	Bus
ER	10	1.00	Bus

The students decide to make a stem-and-leaf plot of the travel times.

A **stem-and-leaf plot** looks like a vertical stem with leaves to the right of it. It is sometimes simply called a *stem plot*.



To make a stem plot to represent travel times, separate each data value into a left "stem" and a right "leaf."

For these data, the "stem" will be the tens digits. Because the travel times include values from 5 minutes to 60 minutes, the stem will be the digits 0, 1, 2, 3, 4, 5, and 6.

- Make a vertical list of the tens digits in order from least to greatest.
- Draw a line to the right of the digits to separate the stem from the "leaves."

0	
1	
2	
3	
4	
5	
6	

The "leaves" will be the ones digits. For each data value, add a leaf next to the appropriate tens digit on the stem.

- The first data value is 60 minutes. Write a 0 next to the stem of 6.
- The next value is 15 minutes. Write a 5 next to the stem of 1.
- The travel times of 30 and 35 minutes are shown by a 0 and 5 next to the stem of 3.

0	
1	5
2	
3	0 5
4	
5	
6	0

Problem 2.1 Making a Stem-and-Leaf Plot

A. Use the Travel to School data to make the stem plot. The plot is started for you.

0	
1	5 5 5 5
2	2 5 0
3	0 5
4	
5	
6	0

B. Now redraw the stem plot, putting the data in each leaf in order from least to greatest. Include a title for your plot. Also include a key like the following that tells how to read the plot.

Key
2 | 5 means 25 minutes

C. Which students probably get to sleep the latest in the morning? Why do you think this?

D. Which students probably get up the earliest? Why do you think this?

E. What is the median of the travel-time data? Explain how you found this.

F. What is the range of the travel-time data? Explain.

ACE Homework starts on page 40.

2.1 Traveling to School

Goals

- Group numerical data in equal intervals and display their distribution using a stem-and-leaf plot
- Find measures of center and variation, including range and how data vary from the least to the greatest values, when a distribution is displayed using a stem-and-leaf plot

Students explore two different contexts (Problems 2.1 and 2.2) in which the data collected are more variable than those in earlier data sets.

Representations such as line plots and bar graphs are not suitable for displaying these data; the patterns within the data sets can only be seen when the data are grouped. The stem-and-leaf plot provides a representational tool that permits grouping data in equal intervals.

This lesson begins to expand the students' ability to look for patterns in data, rather than look at individual data items. A data list or a data table rarely shows where data tend to cluster, nor the gaps between data that can also be important to understanding the complete set.

Launch 2.1

Display Transparencies 2.1A and 2.1B. To engage students in the context of the problem and the data set, refer them to the table of data and the first two questions in the Getting Ready.

Suggested Questions Ask the following and have them work on it for a few minutes before you have a whole class discussion:

- *Look at the table of data. What are the three questions that the students asked to collect this data?* (How long does it take for you to travel to school? How far do you travel to get to school? How do you get to school?)
- *Describe how you think they collected the data to answer each of these questions.* (Possible answer: questionnaire)

Make sure you review with students how the data for distance are in decimal form in multiples of a quarter mile, or 0.25. Students will become more comfortable with the data once they review its format.

Refer students to the third question in the Getting Ready.

- *Would a line plot be a good way to show the travel-time data? Why or why not?*

Help them think about what they would have to do to make a line plot to display these data. You may even want to go through the process of trying to create a line plot. Fairly quickly, students will begin to see that the times these students take to travel to school vary from 5 to 60 minutes. It is difficult to make a line plot numbered 5 minutes, 6 minutes, 7 minutes, and so on, to show exact times. Also, the data are not really clustered by individual times, so it would be difficult to see any patterns in the data. A strategy in which the data are grouped is needed to help us look for patterns in the data.

When students are familiar with the data, introduce the idea of making a stem-and-leaf plot to represent the data.

The Student Edition outlines how to develop stem-and-leaf plots. However, we do not recommend that you have students read through this process on their own. Instead, we encourage you to present the process as a class exploration led by you. Students can consult the Student Edition for reference at a later time. Here is one way you may proceed.

- *Let's build a stem-and-leaf plot. When we look at our data, we see that the travel times vary from 5 minutes to 60 minutes. We can use this information to set up a graph that has a "stem" and several "leaves."*
- *The "leaves" are the units digits of the data values. The other digits in the data values form the stem. In this case, the stem is made up of the tens digits of the travel times.*

Suggested Questions You might ask these questions to review the idea of the tens digit:

- *Suppose a student takes 45 minutes to get to school. What is the tens digit?* (4)
- *What about 15 minutes?* (1)
- *What about 5 minutes?* (0)
- *What is the greatest tens digit we need to show?* (6)

- *We show the tens digits as a "stem" of numbers.*

0	
1	
2	
3	
4	
5	
6	

- *Next, we begin to add "leaves" to the stem by placing each ones digit next to its tens digits. The first student has a travel time of 60 minutes. We show the ones digit (the 0) as a leaf.*

0	
1	
2	
3	
4	
5	
6	0

- *The next travel times are 15 minutes and 30 minutes. How should I add these to the graph?* (Put a 5 by the stem of 1, and 0 by the stem of 3.)

- *The next few travel times are 15 minutes, 15 minutes, 35 minutes, 15 minutes, 22 minutes, 25 minutes, and 20 minutes. Watch how I add these values to the graph.*

- *I would like you to work with a partner to copy this stem-and-leaf plot and to add the remaining leaves.*

0	
1	5 5 5 5
2	2 5 0
3	0 5
4	
5	
6	0

Much of this problem's launch phase focuses on developing an understanding of the stem-and-leaf plot. The first step in this understanding is making a stem plot. Following this, students need to develop a better understanding of the idea that the data are now grouped in intervals and not simply as repeated values of the same measure. For example, 15 minutes is grouped with other data in the interval of 10–19.

Explore 2.1

Students will complete the stem-and-leaf plot in Question A. Notice that the leaves are not in ascending order. They are recorded as they occur in the data list.

Work with students on Question B to rearrange these leaves so they are in order. Transparency 2.1C shows the stem-and-leaf plot before and after arranging the data in ascending order. Help students to add a title to the stem plot and a key for interpreting the plot.

For Questions C and D, remind students as they select the interval to explain their reasoning. For Question E some students may have problems with finding the median. You can help these students by asking how finding the median on the stem-and-leaf plot is similar to finding the median when the data is represented in an ordered list. Check to see that all students are recording their strategies and are ready to explain them. Make note of the strategies you want to have students share in the Summary.

Summarize 2.1

Suggested Questions Ask questions that focus on reading the stem plot and on identifying intervals.

- *What is the shortest time for the 1 stem?* (10 min)
- *What is the longest time for the 1 stem?* (19 min)
- *What possible times are not shown for the 1 stem?* (12 min, 13 min, 14 min, 16 min, and 18 min)
- *We say that the interval of possible times for the 1 stem is from 10 to 19 minutes. What is the interval of possible times for the 0 stem?* (0–9 min)
- *What is the interval of possible times for the 2 stem?* (20–29 min)
- *What is the interval of possible times for the 3 stem?* (30–39 min)

Have pairs share their answers with the whole class. Make sure students explain the reasoning for their answers to Questions C and D. Also, make sure they explain their strategy for finding the median and range in Questions E and F.

The questions in this problem guide students to "read the data" and to "read between the data." Before you leave Problem 2.1, spend some time working with them to "read beyond the data."

- *How can we describe the shape of the data when they are grouped by tens?* (Most of the data cluster in one area of the stem from 0 min to 35 min. There are two outliers: 50 min and 60 min. Both these students take the bus and probably get on at the beginning of the bus route, since they live farther from the school than most of the other students.)
- *Using the mode probably won't tell us too much about the data with this graph. Why do you think this is so? Could we talk about an interval that contains the most data points?* (The mode is the value in the data that occurs most frequently; in these data, the mode is 15 minutes. However, when we look at the stem plot, we are more interested in which interval(s) contain the most values. In these data, when the data are grouped by tens, the interval is 10–19 and it contains 17 values.)
- *How would we find the median for this set of data?* [There are 40 measures in this data set. The median is the number that marks the midpoint between the twentieth and twenty-first values. The second stem plot orders the data, so we can count from either end and locate the twentieth and twenty-first values (15 min and 17 min). Thus the median is 16 min.]

2.1 Traveling to School

At a Glance

PACING 1 day

Mathematical Goals

- Group numerical data in equal intervals and display their distribution using a stem-and-leaf plot
- Find measures of center and variation, including range and how data vary from the least to the greatest values, when a distribution is displayed using a stem-and-leaf plot

Launch

Display Transparencies 2.1A and 2.1B. Refer students to the table of data and the first two questions in the Getting Ready. Have them work on the questions for a few minutes before you have a whole-class discussion.

- *Look at the table of data. What are the three questions that the students asked to collect this data?*
- *Describe how you think they collected the data to answer each of these questions.*

Refer students to the third question in the Getting Ready.

- *Would a line plot be a good way to show the travel-time data? Why or why not?*

When students are familiar with the data, introduce the problem of making a stem-and-leaf plot to represent the data. The Student Edition outlines how to develop stem-and-leaf plots. We do not recommend that you have students read through this process on their own. Instead, we encourage you to present the process as a class exploration led by you. You may need to review the idea of the tens digit.

Materials

- Transparencies 2.1A–C
- Local street map (optional)

Vocabulary

- stem-and-leaf plot

Explore

Students will complete the stem-and-leaf plot in Question A. For Question B, work with students to rearrange these leaves so they are in order. Transparency 2.1C shows the stem and-leaf plot before and after arranging the data in ascending order. Help students add a title and key for the stem plot.

For Questions C and D, remind students as they select the interval to explain their reasoning. For Question E, some students may have trouble finding the median. You can help them by asking how finding the median on the stem-and-leaf plot is similar to finding the median when the data is represented in an ordered list. Check to see that all students are recording their strategies and are ready to explain them. Make note of the strategies you want to have students share in the Summary.

Summarize

Ask questions that focus on reading the stem plot and on identifying intervals.

- *What is the shortest time for the 1 stem?*
- *What possible times are not shown for the 1 stem?*
- *We say that the interval of possible times for the 1 stem is from 10 to 19 minutes. What is the interval of possible times for the 0 stem?*

Have pairs share their answers with the whole class. Make sure students explain their reasoning. Before you leave Problem 2.1, spend some time working with them to "read beyond the data."

- *How can we describe the shape of the data when they are grouped by tens?*
- *Using the mode probably won't tell us too much about the data with this graph. Why do you think this is so? Could we talk about an interval that contains the most data points?*
- *How would we find the median for this set of data?*

Materials

- Student notebooks

ACE Assignment Guide for Problem 2.1

Differentiated Instruction

Core 1–4

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 2.1

A.

0	8 8 5 5 5 6
1	5 5 5 5 9 5 5 7 5 0 5 0 5 1 7 0 0
2	2 5 0 5 0 0 0 0 0 1
3	0 5 0 0 5
4	
5	0
6	0

B. **Student Travel Times to School**

0	5 5 5 6 8 8
1	0 0 0 0 1 5 5 5 5 5 5 5 5 5 7 7 9
2	0 0 0 0 0 0 1 2 5 5
3	0 0 0 5 5
4	
5	0
6	0

Key: 2 | 5 means 25 min

C. Most students will reason that those students who have the shortest travel time (times in the 0–9 min interval) probably sleep the latest. However, because of other variables, times in the 10–19 min interval may be chosen.

D. Students who have longer travel times (the 50 min and 60 min outliers) probably get up the earliest.

E. 16 min; possible explanation: To find the median, count in from both ends of the plot until you reach the midpoint. It is between 15 and 17 min, so the median is 16 min.

F. 55 min; possible explanation: The range is the difference between the greatest value on the plot and the least value: $60 - 5 = 55$.

2.2 Jumping Rope

Mrs. Reid's class competed against Mr. Costo's class in a jump-rope contest. Each student jumped as many times as possible. Another student counted the jumps and recorded the total. The classes made the *back-to-back stem plot* shown to display their data. Look at this plot carefully. Try to figure out how to read it.

When the two classes compare their results, they disagree about which class did better.

- Mr. Costo's class says that the range of their data is much greater.
- Mrs. Reid's class says this is only because they had one person who jumped many more times than anybody else.
- Mrs. Reid's class claims that most of them jumped more times than most of the students in Mr. Costo's class.
- Mr. Costo's class argues that even if they do not count the person with 300 jumps, they still did better.

Number of Jumps

Mrs. Reid's class		Mr. Costo's class
8 7 7 7 5 1 1	0	1 1 2 3 4 5 8 8
6 1 1	1	0 7
9 7 6 3 0 0	2	3 7 8
7 6 5 3	3	0 3 5
5 0	4	2 7 8
	5	0 2 3
2	6	0 8
	7	
9 8 0	8	
6 3 1	9	
	10	2 4
3	11	
	12	
	13	
	14	
	15	1
	16	0 0
	17	
	18	
	19	
	20	
	21	
	22	
	23	
	24	
	25	
	26	
	27	
	28	
	29	
	30	0

Key: 7 | 3 | 0 means 37 jumps for Mrs. Reid's class and 30 jumps for Mr. Costo's class

Problem 2.2 Comparing Distributions

A. Which class did better overall in the jump-rope contest? Use what you know about statistics to help you justify your answer.

B. In Mr. Costo's class, there are some very large numbers of jumps. For example, one student jumped 151 times, and another student jumped 300 times. We call these data outliers. **Outliers** are data values that are located far from the rest of the other values in a set of data. Find two other outliers in the data for Mr. Costo's class.

C. An outlier may be a value that was recorded incorrectly, or it may be a signal that something special is happening. All the values recorded for Mr. Costo's class are correct. What might account for the few students who jumped many more times than their classmates?

ACE **Homework starts on page 40.**

2.2 Jumping Rope

Goals

- Compare two distributions displayed using back-to-back stem-and-leaf plots
- Compare two distributions using statistics, such as median, range, and how the data vary from least to greatest values
- Identify outliers in a distribution

You may choose to investigate the problem using the data provided, or you may want to help your students conduct their own jump-rope activity and collect their own data. If your class conducts the activity, you will need to develop procedures for collecting the data. (Be aware that collecting these data is time-consuming!) You might ask the physical education teacher to help your students collect the data during their physical education class.

You may want to explore the problem using the data presented in the Student Edition and then extend the exploration phase to include your students' data, making comparisons where appropriate.

Launch 2.2

Present the problem by using Transparency 2.2 or by referring students to the Student Edition. Work with your students to make sure they can read the back-to-back stem plot before they begin to work on the problem. One way to do this is to cover the left side of the stem plot and ask students what information is shown on just the right side. Then cover the right side, and have students discuss how the data on the left side are read (when the stem is on the right). Finally, you can show both sets of data together, discussing how this arrangement lets you make comparisons between data sets.

Have students work in pairs or small groups.

Explore 2.2

Once students are comfortable with the data display, they can focus on the questions posed in Problem 2.2.

Suggested Questions For students who need support in answering Question A, ask:

- *What statistic might help you compare the class's jump roping?* (If necessary, remind them of the statistics that they talked about in Problem 2.1. For each statistic they mention, ask them why they chose this statistic and what it might tell them about the data. Try to have them realize that the median, how the data vary from the least to the greatest values, and range are very useful to compare data, and the mode may not be as useful.
- *How would you find that statistic using the data?* (Depending on the statistic they choose, you will need to ask them questions to help find the specific statistic they chose.)

Summarize 2.2

Hold a class discussion about Problem 2.2. The process of comparison may be difficult. Students may wonder how they can compare data sets that contain different numbers of data items. They may not immediately think about finding the medians, the least and greatest data values, and the ranges of the two classes' data, yet these are precisely the tools that can help them make comparative statements.

At a Glance

2.2 Jumping Rope

PACING 1 day

Mathematical Goals

- Compare two distributions displayed using back-to-back stem-and-leaf plots
- Compare two distributions using statistics, such as median, range, and how the data vary from least to greatest values
- Identify outliers in a distribution

Launch

Present the problem by using Transparency 2.2 or by referring students to the Student Edition. Work with your students to make sure they can read the back-to-back stem plot before they begin to work on the problem. One way to do this is to cover the left side of the stem plot and ask students what information is shown on just the right side. Then cover the right side, and have students discuss how the data on the left side are read (when the stem is on the right). Finally, you can show both sets of data together, discussing how this arrangement lets you make comparisons between data sets.

Have students work in pairs or small groups.

Materials
- Transparency 2.2

Vocabulary
- outlier

Explore

Once students are comfortable with the data display, they can focus on the questions posed in Problem 2.2.

For students who need support in answering Question A, ask:

- *What statistic might help you compare the class's jump roping?*
- *How would you find that statistic using the data?*

Summarize

Hold a class discussion about Problem 2.2. The process of comparison may be difficult. Students may wonder how they can compare data sets that contain different numbers of data items. They may not immediately think about finding the medians, the least and greatest values, and the ranges of the two classes' data, yet these are precisely the tools that can help them make comparative statements.

Materials
- Student notebooks

ACE Assignment Guide for Problem 2.2

Differentiated Instruction

Core 5–7, 10, 13

Other *Extensions* 14; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 2.2

A. There are a variety of ways students can respond to this question. They may compute and compare the medians and the ranges, discuss the presence of outliers, or describe the shape of the data. The median number of jumps for Mrs. Reid's class is 28, with a range of $113 - 1 = 112$ jumps. The median number of jumps for Mr. Costo's class is 34, with a range of $300 - 1 = 299$ jumps. If the four outliers in Mr. Costo's class are ignored, the median number of jumps is 29, with a range of $104 - 1 = 103$ jumps. In the end, students need to give a well-developed response that makes their reasoning clear.

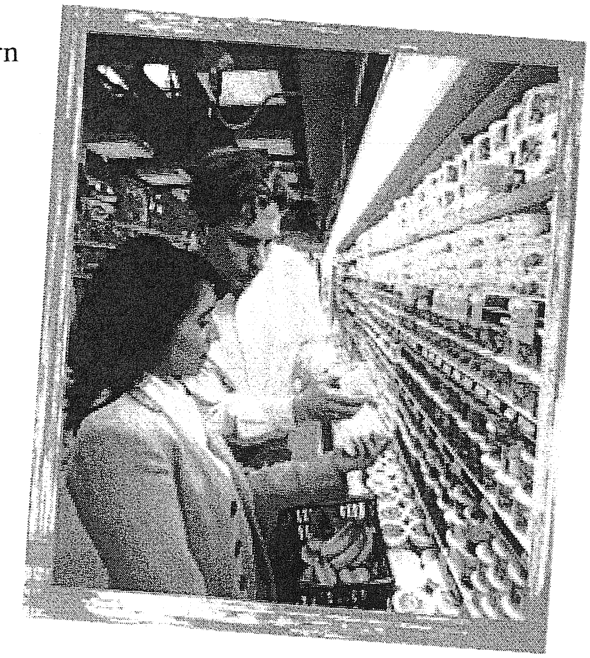
B. The two other outliers in Mr. Costo's class are both 160 jumps.

C. Possible answers: The students are physically fit, or the students jump rope frequently.

Investigation 1

Comparing Data Sets

American shoppers have a great variety of products from which to choose. Many people turn to information in consumer surveys and product comparisons to help make decisions.



A consumer magazine rated 37 varieties of peanut butter. Each peanut butter was assigned a quality rating from 1 to 100 points. A panel of trained tasters made two general statements about quality:

- Peanut butters with higher quality ratings were smooth; had a sweet, nutty flavor; and were not overly dry or sticky.
- Peanut butters with lower quality ratings were not very nutty, had small bits of peanuts, or had a burnt or slightly rancid taste.

The article also gave the sodium content and price per 3-tablespoon serving for each type. Peanut butters were classified according to three attributes: natural or regular, creamy or chunky, and salted or unsalted. The data are presented in the table on the next page. A fourth attribute, name brand or store brand, has been added to the data.

Peanut Butter Comparison

	Peanut Butter	Quality Rating	Sodium per Serving (mg)	Price per Serving (cents)	Regular/ Natural	Creamy/ Chunky	Salted/ Unsalted	Name Brand/ Store Brand
1.	Smucker's Natural	71	15	27	natural	creamy	unsalted	name
2.	Deaf Smith Arrowhead	69	0	32	natural	creamy	unsalted	name
3.	Adams 100% Natural	60	0	26	natural	creamy	unsalted	name
4.	Adams	60	168	26	natural	creamy	salted	name
5.	Laura Scudder's All Natural	57	165	26	natural	creamy	salted	name
6.	Country Pure Brand	52	225	21	natural	creamy	salted	store
7.	Hollywood Natural	34	15	32	natural	creamy	unsalted	name
8.	Smucker's Natural	89	15	27	natural	chunky	unsalted	name
9.	Adams 100% Natural	69	0	26	natural	chunky	unsalted	name
10.	Deaf Smith Arrowhead	69	0	32	natural	chunky	unsalted	name
11.	Country Pure Brand	67	105	21	natural	chunky	salted	store
12.	Laura Scudder's All Natural	63	165	24	natural	chunky	salted	name
13.	Smucker's Natural	57	188	26	natural	chunky	salted	name
14.	Health Valley 100%	40	3	34	natural	chunky	unsalted	name
15.	Jif	76	220	22	regular	creamy	salted	name
16.	Skippy	60	225	19	regular	creamy	salted	name
17.	Kroger	54	240	14	regular	creamy	salted	store
18.	NuMade	43	187	20	regular	creamy	salted	store
19.	Peter Pan	40	225	21	regular	creamy	salted	name
20.	Peter Pan	35	3	22	regular	creamy	unsalted	name
21.	A & P	34	225	12	regular	creamy	salted	store
22.	Food Club	33	225	17	regular	creamy	salted	store
23.	Pathmark	31	255	9	regular	creamy	salted	store
24.	Lady Lee	23	225	16	regular	creamy	salted	store
25.	Albertsons	23	225	17	regular	creamy	salted	store
26.	ShurFine	11	225	16	regular	creamy	salted	store
27.	Jif	83	162	23	regular	chunky	salted	name
28.	Skippy	83	211	21	regular	chunky	salted	name
29.	Food Club	54	195	17	regular	chunky	salted	store
30.	Kroger	49	255	14	regular	chunky	salted	store
31.	A & P	46	225	11	regular	chunky	salted	store
32.	Peter Pan	45	180	22	regular	chunky	salted	name
33.	NuMade	40	208	21	regular	chunky	salted	store
34.	Lady Lee	34	225	16	regular	chunky	salted	store
35.	Albertsons	31	225	17	regular	chunky	salted	store
36.	Pathmark	29	210	9	regular	chunky	salted	store
37.	ShurFine	26	195	16	regular	chunky	salted	store

SOURCE: *Consumer Reports* and *Workshop Statistics: Student Activity Guide*

Getting Ready for Problem 1.1

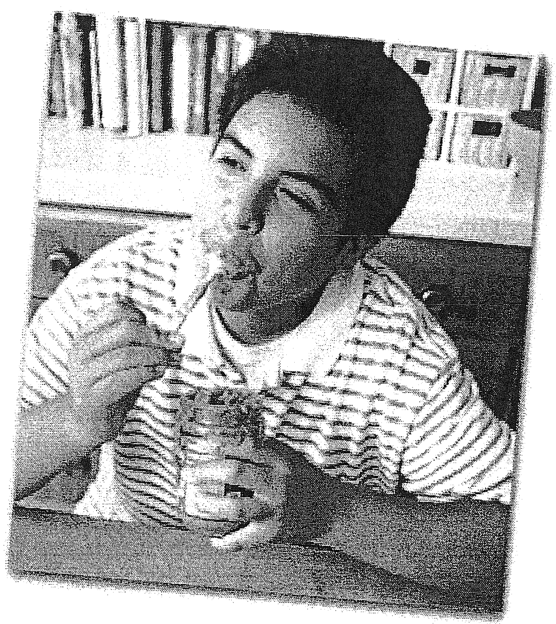
- Who might be interested in the results of this peanut butter study?
- What questions about peanut butter can be answered with these data?
- What questions about peanut butter cannot be answered with these data?

1.1 From Line Plots to Histograms

In this problem, you will look at the **distribution** of quality ratings for the regular peanut butters. You will use measures of center, minimum and maximum values, range, the shape of the data, and where the data cluster to describe the distribution. Locate quality ratings in the table.

Did You Know?

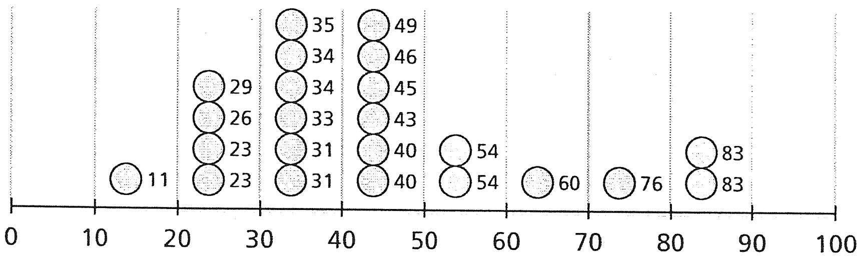
Arachibutyrophobia (uh rak ih byoo tih ruh FOH bee uh) is the fear of getting peanut butter stuck to the roof of your mouth!



Problem 1.1 From Line Plots to Histograms

A. Each dot on the line plot below represents the quality rating of one regular peanut butter from the table.

Regular Peanut Butter Quality Ratings

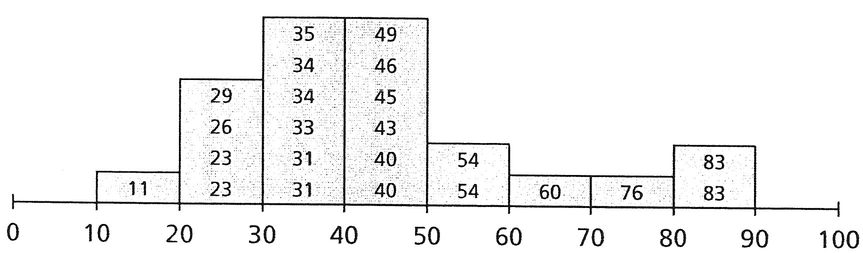


Quality Rating

1. Which interval (or intervals) includes the most quality ratings?
2. Look at the interval marked 40 to 50. What is the lowest rating in this interval? What is the highest rating in this interval?
3. Suppose you want to add a quality rating of 50 to the plot. In which interval would you put this value? Explain.
4. Suppose you want to add a quality rating of 59. In which interval would you put this value? Explain.
5. What do you think is the typical rating for regular peanut butters? Explain.

B. In the plot below, the collection of dots in the intervals have been used to make bars that show the number of data values in each interval.

Regular Peanut Butter Quality Ratings



Quality Rating

1. To which interval would you add each of these quality ratings: 93, 69, 10, and 57?
2. How would you change the bar in an interval to show the addition of a new quality rating?

C. The **histogram** below shows the same distribution as the interval bars with numerical values in Question B. A frequency axis has been added to the side of the plot.

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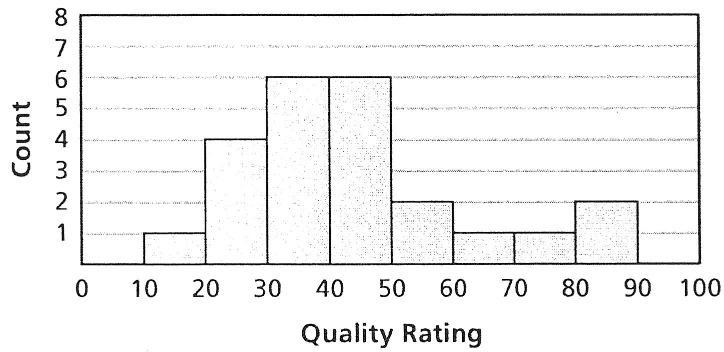
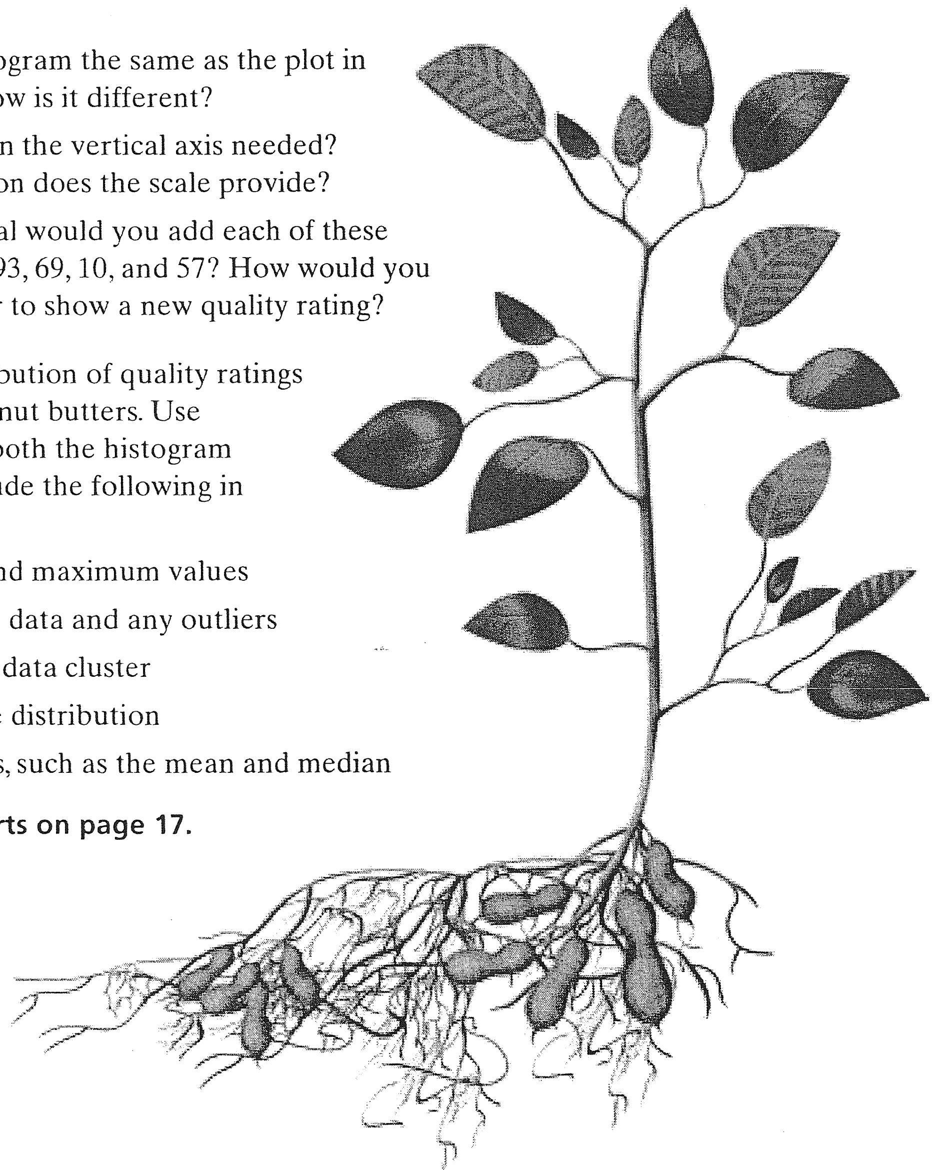
Regular Peanut Butter Quality Ratings



1. How is this histogram the same as the plot in Question B? How is it different?
2. Why is a scale on the vertical axis needed? What information does the scale provide?
3. To which interval would you add each of these quality ratings: 93, 69, 10, and 57? How would you change each bar to show a new quality rating?

D. Describe the distribution of quality ratings for the regular peanut butters. Use information from both the histogram and the table. Include the following in your description:

- the minimum and maximum values
- the range of the data and any outliers
- intervals where data cluster
- the shape of the distribution
- related statistics, such as the mean and median



ACE **Homework starts on page 17.**

1.1 From Line Plots to Histograms

Goal

- Compare sample distributions using measures of center (mean, median), measures of variability (minimum and maximum values, range), and data displays that group data (histograms)

In Problem 1.1, students explore the quality ratings of regular peanut butters. They learn how to make histograms to represent these data.

Launch 1.1

Students use one data set throughout Investigation 1. From a consumer-product study, the data consist of information about the quality, sodium content, and price of 37 types of peanut butter classified by four attributes: natural or regular (type), creamy or chunky (consistency), salted or unsalted (salt content), and name brand or store brand (distributor).

One optional way to introduce this investigation is to engage students in a simple taste test of two varieties of peanut butter.[1] If possible, choose peanut butters from the table of data in the student edition, selecting one with a low quality rating (perhaps a store brand from your local supermarket) and one with a high quality rating (for example, Jif). Make sure to check whether any students have an allergy to peanuts before you do this activity.

To conduct the taste test remove the labels from the jars. Label one jar A and the other jar B. Prepare, or have a student prepare, enough crackers with peanut butter A and enough crackers with peanut butter B for each student to have one cracker for each type of peanut butter. (This is a good opportunity to discuss important criteria for taste tests, such as using the same type of cracker for each peanut butter and spreading the same amount of peanut butter on each cracker.)

Have students take one of each cracker at the same time to do their taste tests. (Teachers have found that students need to taste the peanut butters at the same time to be able to make comparisons. Also, you may want to have half your class taste A first followed by B and the other half do the alternative order. They can drink a little water between tests to "clean their palates.")

Ask students to rate the taste of each product on a scale of 1 to 10, with 10 being the highest.

Students' ratings for each of the two peanut butters can be displayed on two separate line plots, each scaled from 1 to 10. Once the two line plots are displayed, ask students to discuss which type is better based on their ratings. Use this as an opportunity to help the class consider what kind of criteria might be used to determine a quality rating for a particular type of peanut butter and to help students realize how similar their ratings may be to one another. Ratings will likely cluster in one section of each line plot. Students are not trained tasters, yet they share a sense of how good a peanut butter tastes.

First, use Transparencies 1.1A and 1.1B to have a conversation with students about the data and about questions that the data might help to answer. In considering the questions, students will become familiar with the table of data.

Suggested Questions

- *Who might be interested in the results of this study?* (Peanut butter is a common food in many households; these data may be of interest to anyone who eats peanut butter. With the increasing attention being paid to healthful diets, such nutritional information is important. Ask the class who might be the audience for Consumer Reports, the magazine in which the initial study was reported. Many people have an interest in comparative shopping; this magazine presents such data so that consumers may make more informed purchasing decisions.)
- *What interesting questions about peanut butter can be answered using these data?*

[1] For a more detailed discussion, see Friel, S. N., & O'Connor, W. (1998) Sticks to the roof of your mouth. *Mathematics Teaching in the Middle School, 4*, 404-411.

A few of the questions that can be asked about types of peanut butter guide students' work in this investigation, but many other questions are possible. Here are a few questions that can be addressed by analyzing this set of data:

- *Is there a lot of salt in peanut butter?*
- *Is there much variability in quality ratings among different kinds of peanut butter?*
- *What's the best buy if I am most interested in quality?*
- *What's the best buy if I am most interested in price?*
- *I like Jif chunky peanut butter. How does it stack up against the other peanut butters in this study?*
- *What interesting questions about peanut butter can't be answered using these data?*

Here a few questions that might be asked but that cannot be addressed by analyzing this set of data:

- *Who eats peanut butter?*
- *What preferences for peanut butter do students in our class have?*
- *Does peanut butter contain a high amount of fat? Do different types differ widely in their fat content?* (Information on fat content is not shown in the table. These questions might make an interesting research topic, especially with reduced-fat peanut butters now on the market.)
- *Which types shown in the table are sold in our local grocery stores?*
- *If we conducted our own taste test, trying some of the higher-rated peanut butters and some of the lower-rated peanut butters, would our results agree with these?*

Use Transparencies 1.1C and D to work through the development of how histograms can be made. Some of these graphs are displayed in the students' edition so that students will have a way to remember how a histogram is made. Ask students to explain the relationship between these three graphs.

- *How is the interval-stacked line plot developed from the shaded-intervals line plot?*

On Transparencies 1.1C and D, a line plot is shown first. Talk with students about how this graph is related to the data about quality ratings found in the table. What do they notice about (1) the overall shape (e.g., where do the data cluster, how are the data spread out, is the shape more mounded or skewed); (2) how the data vary from the minimum to the maximum data values and what is the range; (3) where the mean or the median might be located.

Next the distribution is shown divided into ten intervals. From this, a grouped-data line plot is created that shows the data stacked and ordered in equal width intervals of 10. Ask students to explain the relationship between these two graphs. How is the grouped-data line plot developed using the data shown in the line plot?

Data are fused or merged together to form *bars*. Finally, the actual data values are removed to focus on the shape of the histogram. Ask students how this graph relates to the previous grouped-data line plot. Tell students that this graph is called a histogram.

Have the students work in pairs and go through Problem 1.1. The problem will give students opportunities to make more sense of the ideas you introduced in the Launch. Students need this chance to go back and work though the points made earlier.

Explore 1.1

Students take more time and look at the detail of histograms and how data are organized. They should be able to work though the questions.

Summarize 1.1

The main points about the structure of a histogram should be discussed: how are data organized once an interval width is chosen and what happens to *border* data points—in which interval is a border data point placed?

1.1 From Line Plots to Histograms

At a Glance

PACING 1 day

Mathematical Goal

- Compare sample distributions using measures of center (mean, median), measures of variability (minimum and maximum values, range), and data displays that group data (histograms)

Launch

Read the introduction up to Getting Ready. Then put up Transparency 1.1A. See the extended Launch for a discussion of a peanut butter taste test that you may want to have your students do. Distribute Labsheet 1.1 and have the students look at these data. Ask them to consider the questions in the Getting Ready.

Problem 1.1 begins with an introduction to histograms by asking students to apply their knowledge of data analysis to compare quality ratings for natural and regular brands of peanut butters. The comparison helps them to determine the category of peanut butter having an overall higher quality rating. Use Transparencies 1.1C and D to work through the development of how histograms can be made. Some graphs are in the students' edition. Ask students to explain the relationship between these three graphs.

- *How is the interval stacked line plot developed from the shaded-intervals line plot?*

Ask students to think about what is alike or different in the two versions of the data in the merged intervals plot and the histogram. Have students work on the problem in pairs.

Materials

- Transparencies 1.1A–D
- Labsheet 1.1
- Graphing calculator
- Computers and statistical software (optional)

Vocabulary

- distribution

Explore

As you circulate, you may need to review strategies for making histograms and locating specific data values.

Summarize

The main points about the structure of a histogram should be discussed: how are data organized once an interval width is chosen and what happens to *border* data points—in which interval is a border data point placed?

Materials

- Student notebooks
- Overhead graphing calculator

ACE Assignment Guide for Problem 1.1

Differentiated Instruction

Core 1
Other *Connections* 27–28

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 27–28: *Data About Us*

Answers to Problem 1.1

A. 1. Intervals 30 to 40 and 40 to 50 each have 6 data values.

2. Smallest data value is 40; largest data value is 49.

3. In the interval of 50 to 60 because a *border point* data value is moved up to the next interval. Students may observe that the data point 40 is in the 40 to 50 interval so it makes sense to put 50 in the 50 to 60 interval.

4. In the interval of 50 to 60 because 59 is still less than 60, the start of the next interval.

5. The typical rating is around 40 (between 30 and 50).

B. 1. 93 would be added to the interval 90 to 100; 69 would be added to the interval 60 to 70; 10 would be added to the interval 10 to 20; 57 would be added to the interval 50 to 60.

2. Increase the height of the bar to show the addition of one more data value; it really doesn't matter if the data values are ordered but students may say that, with ordered data, the data value would be inserted in its correct location and the result is that the height of the bar is increased by 1 more unit.

C. 1. It has the same shape and the same intervals and the same vertical axis labeled in the same way. It does not show the individual data values marked in the intervals; we cannot identify exact values from the graph for data that are located in each interval.

2. The vertical axis provides information about frequency, that is, how many data values occur, say, in the interval of 40 to 50.

Note: If the vertical axis is reported in percents, then it provides information about what percent of all the data values are found, say, in the interval of 40 to 50.

3. As was noted in Question B, part (1), 93 would be added to 90 to 100; 69 would be added to 60 to 70; 10 would be added to 10 to 20; 57 would be added to 50 to 60. The heights of the bars would be increased accordingly.

D. The data vary from 11 to 83 quality points and the range is 72 quality points which is quite large given that the possible quality points has a range of 100 points. There are some high and low quality ratings that may be outliers. The data cluster in the intervals of 20 to 50 quality points; the distribution is mound shaped with a bit of a tail going out toward the higher quality ratings. The median is 40 and the mean is 42.7391 quality points; the mean is about 2 points greater than the median that suggests that there are some higher values influencing the value of the mean.

1.2 Using Histograms

In this problem, you will consider quality ratings for the natural peanut butters, which have no preservatives. By comparing histograms, you can decide whether natural or regular peanut butters have higher quality ratings.

Getting Ready for Problem 1.2

This list summarizes how to examine and describe a data distribution.

- *Read the data.* Identify individual values, the minimum and maximum data values, and the range.
- *Read between the data.* Identify intervals where the data cluster or there are gaps in the data.
- *Read beyond the data.* Describe the shape of the distribution. Identify statistics, such as the mean and median, and relate them to the shape of the distribution.

Look back at the way you described the distribution of quality ratings in Problem 1.1. Did you consider all the things mentioned above?

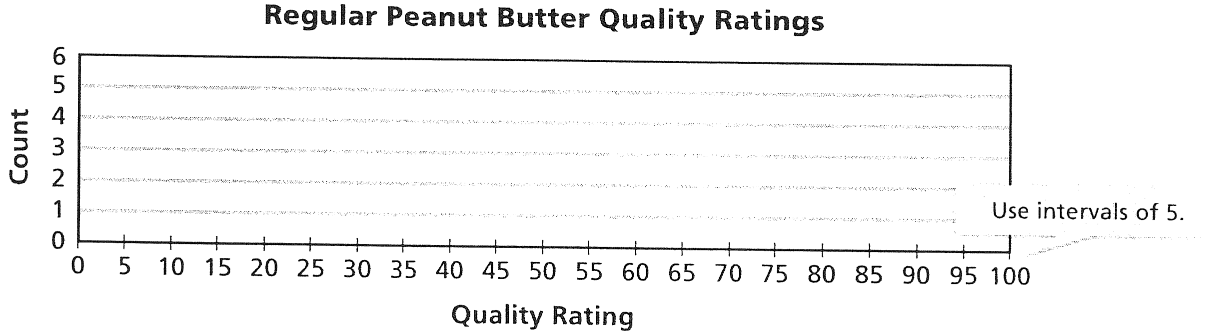
Problem 1.2 Using Histograms

A. 1. Make a histogram of the quality ratings for natural peanut butters. Use 10-point interval widths.

2. Describe the distribution of quality ratings.

B. 1. The histogram in Problem 1.1 shows the quality ratings for the regular peanut butters. The interval width is 10 quality points. Make a new histogram of the quality ratings for regular peanut butters. This time, use interval widths of 5.

Regular Peanut Butter Quality Ratings

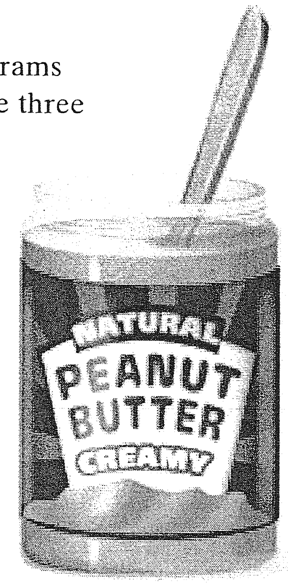


2. Make another histogram of the same data. Use interval widths of either 2 or 15.

3. a. Compare the histogram from Problem 1.1 and the histograms you made in parts (1) and (2). What is the same about the three histograms? What is different?

   b. What are the reasons for the differences in the histograms?

   c. Would your decision about what is a typical quality rating be affected by the histogram you used? Explain.



4. This rule of thumb can help you choose a good interval width for a histogram:

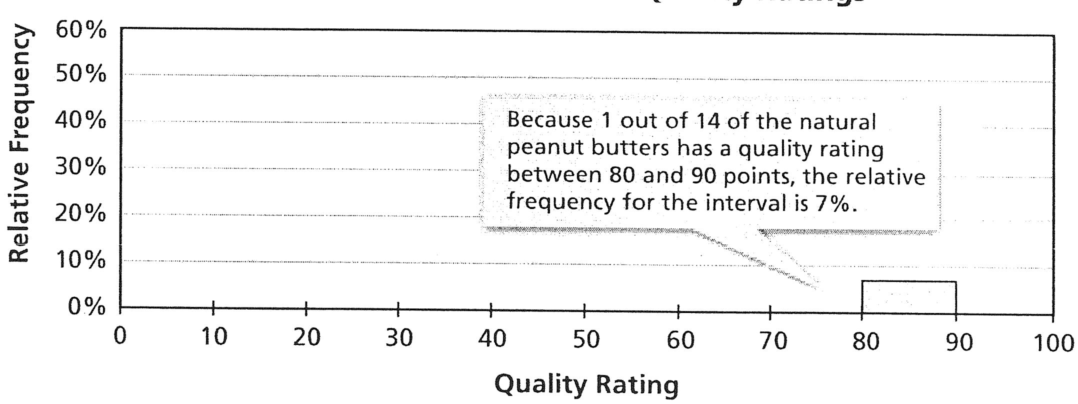
   If possible, use a width that gives 8–10 bars.

   Using this rule of thumb, which of the three histograms is best for representing the distribution of quality ratings for the regular peanut butters?

C. When the data sets you want to compare have different numbers of entries, you can change the vertical axis to show the percent of all values that lie in each interval. This is called a *relative frequency histogram*.

1. Make two new histograms like the one started below, one for natural peanut butters and one for regular peanut butters. Because 1 out of 14 of the natural peanut butters has a quality rating between 80 and 90 points, the relative frequency for this interval is 7%.

Natural Peanut Butter Quality Ratings



2. Do natural peanut butters or regular peanut butters have higher quality ratings? Use the histograms and other relevant information to justify your choice.

ACE Homework starts on page 17.

1.2 Using Histograms

Goal

- Compare sample distributions using measures of center (mean, median), measures of variability (range, percentiles), and data displays that group data (histograms).

In Problem 1.2, students explore using histograms and whatever else they already know about data analysis to compare the quality ratings for natural and regular peanut butters.

Use the questions in the Getting Ready, Transparency 1.2A, to help students look back at their work with graphing data about quality ratings for regular peanut butters from Problem 1.1. This also provides a review of how the data vary from the least and greatest values, range, outliers, mean, and median. The three sections are organized so that students have strategies for thinking about what it might mean when they are asked to describe the distribution of data.

Read the data: Students first focus on the individual data cases. The data vary from 11 to 83 quality points, the range is 72 quality points, and there do not appear to be outliers.

Read between the data: Next, students focus on groups of data. The data cluster in the interval of about 20 to 50 quality ratings; if we draw a vertical line on the graph on Transparency 1.2B at 50, then 26% of the regular peanut butter ratings would be above this quality rating. Drawing a line at 40 places 52% above the quality rating of 40.

Read beyond the data: Finally, students look at other ways to characterize the data. The distribution is mound-shaped, with data clustered in one main area. The median is 40 and the mean is 42.7391 or about 43 points.

You can talk with the students about how knowing these *facts* provides a description of the distribution that lets them understand how the data are related.

Launch 1.2

Problem 1.2 asks students to continue to explore the structure of histograms. Its also questions them to apply their knowledge of data analysis to compare quality ratings for natural and regular peanut butters and to draw conclusions about which category of peanut butter has an overall higher quality rating. Make sure students understand the two parts of the task.

Before students explore Question B, you may want to ask them if they could describe a process for making a histogram that displays data grouped in intervals of 5 (not 10 as was done earlier). Use quality ratings for regular peanut butter. These data are shown below in order from smallest to largest.

11, 23, 23, 26, 29, 31, 31, 33, 34, 34, 35, 40, 40, 43, 45, 46, 49, 54, 54, 60, 76, 83, 83

Consider what it means to group data in intervals of 5 quality points.

Suggested Questions (Use Transparency 1.2B)

- *What would be the first interval?* (10–15)
- *What data values would be in the interval of 10 to 15?* (11)
- *What data values would be in the interval of 15 to 20?* (none)
- *What data values would be in the interval of 20 to 25?* (23, 23)

Show students how to make a frequency table:

Frequency Table

Frequency Interval	What Data?	Number of Data Values
10–15	11	1
15–20		0
20–25	23, 23	2
25–30	26, 29	2
30–35	31, 31, 33, 34, 34	5
35–40	35	1
40–45	40, 40, 43	3
45–50	45, 46, 49	3
50–55	54, 54	2
55–60		0
60–65	60	1
65–70		0
70–75		0
75–80	76	1
80–85	83, 83	2

The question of where to locate the quality rating of 35 needs to be addressed (i.e., in which interval). Remind students that we are making a graph that is built using a number line.

In a line plot, the data value would be marked at 35. In a histogram that displays data values grouped in intervals, we need to know what *convention* is used to decide which of the two intervals that are possible (30 to 35 or 35 to 40) is the one to choose. We choose 35 to 40.

In the interval of 30 to 35, all data values that are $30 \leq$ data value < 35 are included.

In the interval of 35 to 40, all data values that are $35 \leq$ data value < 40 are included.

The data value, 35, is placed in the 35 to 40 interval.

When students get to B they can continue to fill in the chart (Transparency 1.2B). Ask students to relate the frequency table to the structure of their histogram. (Figure 1)

Have students work in pairs.

Explore 1.2

As you circulate, you may need to help the students prepare different histograms about the data for quality ratings for regular peanut butters. You also may need to help students review the distinction between using a frequency scale that is reported as counts and one that is reported as relative frequencies (percents).

Summarize 1.2

Have pairs of students report their findings. Ask for justification for the conclusions they reach.

Use the information reported to review the statistics of the mean, the median, and the range.

Natural vs. Regular Peanut Butter

	Natural Peanut Butters	Regular Peanut Butters
Mean	61.2	42.7
Median	61.5	40
Range	34 to 89	11 to 83

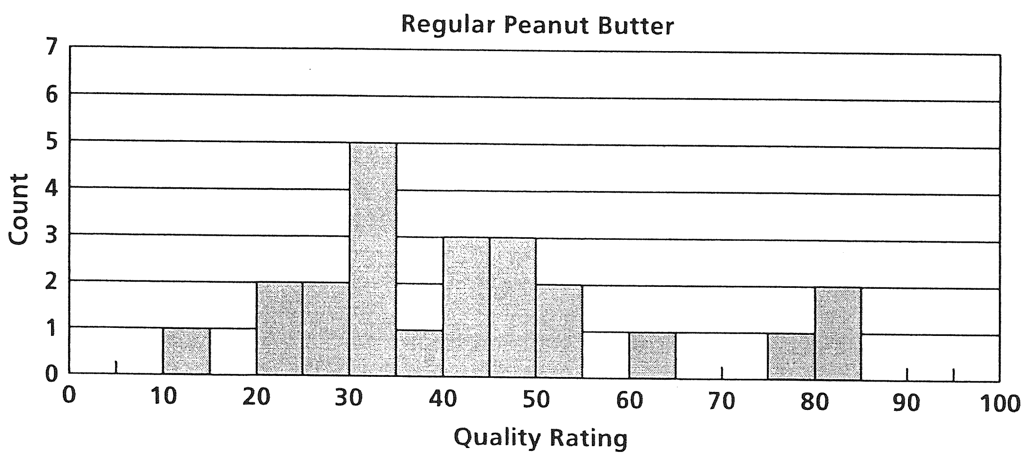
Involve students in a discussion about what information they can see with each graph and how the graph helps or hinders making comparisons. With histograms, the shape of the data may be discussed; including the range and any clusters, gaps, peaks, and tails (or skewness).

Ask students to think about other questions or comparisons that might be interesting to investigate with a different analysis of the data.

Review median and the mean as two ways to summarize data. The median as a value is considered to be more stable than the mean; it always marks the midpoint in a distribution with the same number of data values occurring before and after the median. Review the idea that if the mean is greater than the median, there are some unusually high values in the data; if the mean is less than the median, there are some unusually low values in the data.

Spend time distinguishing between counts as frequencies and percents as frequencies. Consider the reason for using relative frequencies: when two data sets have unequal numbers of values, then making distributions using relative frequencies permits comparing the data sets as though they were equal in numbers of values.

Figure 1



1.2 Using Histograms

At a Glance

PACING 1 day

Mathematical Goal

- Compare samples using measures of center (mean, median), measures of variability (range, percentiles), and data displays that group data (histograms)

Launch

Discuss the questions in the Getting Ready. Then move to working on Problem 1.2.

Ask students if they could describe a process for making a histogram that displays data grouped in intervals of 5. Use quality ratings for regular peanut butter.

- *What would be the first interval?*
- *What numbers would be in the interval of 10 to 15?*
- *What numbers would be in the interval of 15 to 20?*
- *What numbers would be in the interval of 20 to 25?*

Show students how to make a frequency table. See the extended Launch for table and discussion. Continue with the students to fill in the chart. Then display the histogram, asking students to relate the frequency table to the structure of the histogram. Have students work on the problem in pairs.

Materials

- Transparencies 1.2A–C
- Labsheet 1.1
- Graphing calculator
- Grid paper

Explore

As you circulate, you may need to review strategies for finding the median, mean, and range. You also may need to help students review the distinction between using a frequency scale that is reported as counts and one that is reported as relative frequencies (percents).

Summarize

Have pairs of students report and justify their findings and conclusions. Use the information reported to review the statistics of the mean, the median, and the range. Discuss with students the information they see with each graph and how each type of graph helps or hinders making comparisons. Ask students to think of other questions or comparisons interesting to investigate using a different analysis of the data. Review median and mean as two ways to summarize data. Spend time distinguishing between counts and percents as frequencies, and discuss the reason for using relative frequencies.

Materials

- Student notebooks
- Overhead graphing calculator

ACE Assignment Guide for Problem 1.2

Differentiated Instruction

Core 2–6, 29

Other unassigned choices from previous problems

Adapted For suggestions about adapting Exercises 3–6 and other ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 1.2

A. 1. (Figure 2)

2. The distribution of natural peanut butters shows the quality ratings between 34 and 89 and a range of 55 points, with no gaps and a cluster between 50 and 70 (more falling between 60 and 70 than 50 and 60). 64% of the data have 60 or higher quality ratings.

B. 1. (Figure 3 next page)

2. (Figures 4 and 5, next page)

3. a. They all have similar range, mean, and median. However, the larger the interval width, the less precision there is—for example, where clusters and gaps occur.

b. The larger interval widths group together more data values than smaller interval widths. This gives us less information about the exact data values. The smallest interval widths group very few data values together, which may make overall patterns more difficult to see.

c. It's easier to see what is "typical" if the histogram is mound-shaped or has a clear cluster of data values. You can see what is typical from all of the graphs except the 2–interval graph. The 10-interval graph gives both the overall shape and precision. We might choose a lower "typical" value for the 5-interval graph because the shape is less clear. We might select a wider interval, 30–45, for what is "typical" of the 15-interval graph.

4. The histogram with an interval of 10 is the best choice using this rule of thumb.

C. 1. (Figures 6 and 7, page 30)

2. In comparing the two kinds of peanut butters, it looks like natural peanut butters have higher ratings overall; these data are clustered around 50–70 while the regular data are clustered around 30–50.

Figure 2

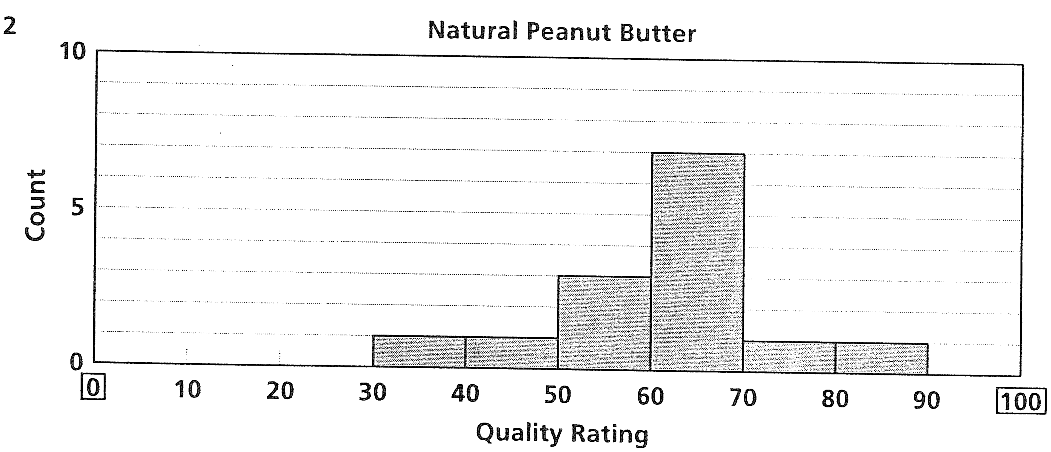


Figure 3

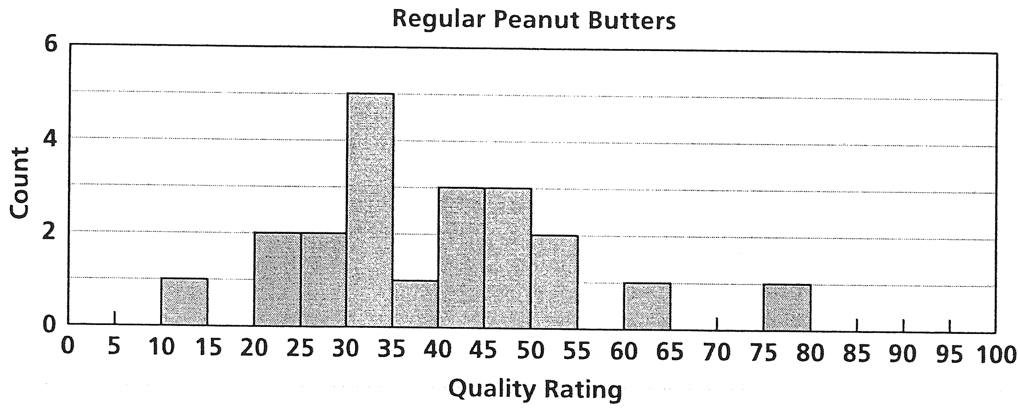


Figure 4

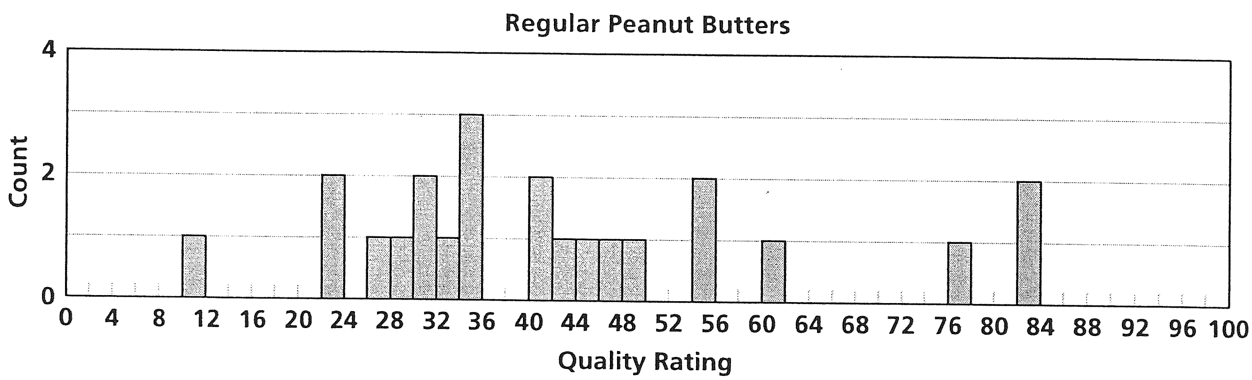


Figure 5

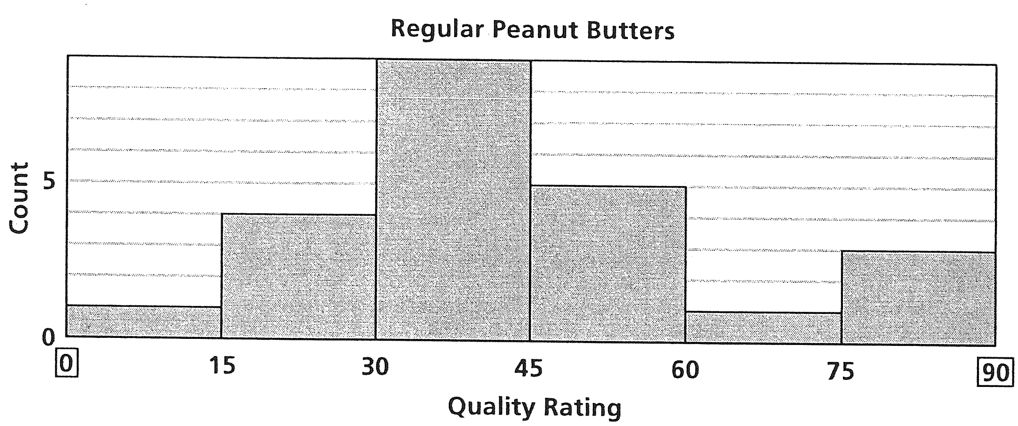


Figure 6

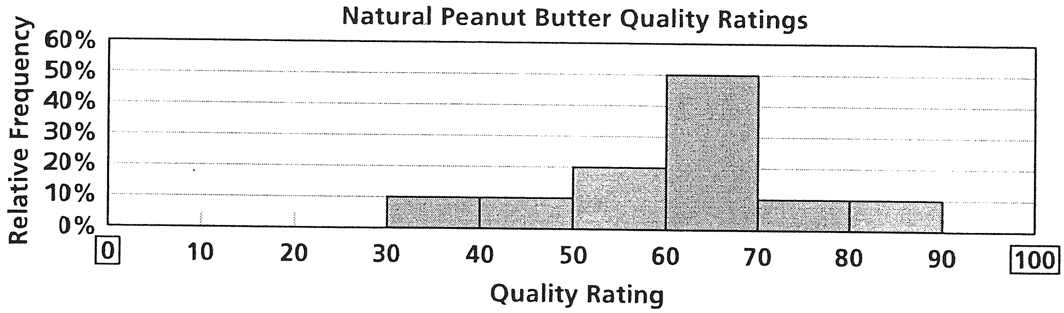
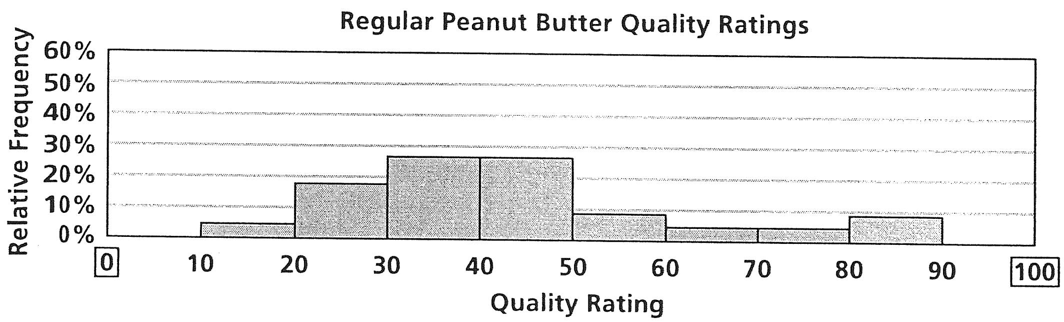


Figure 7

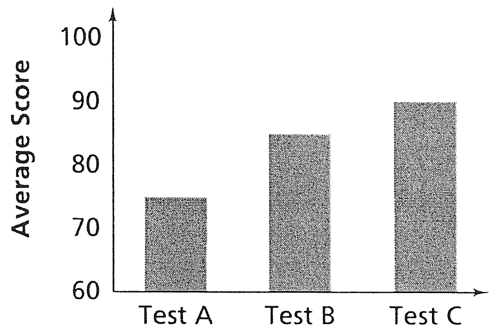


Topic 10: Misleading Data Displays

A graph can sometimes give a misleading picture of the data.

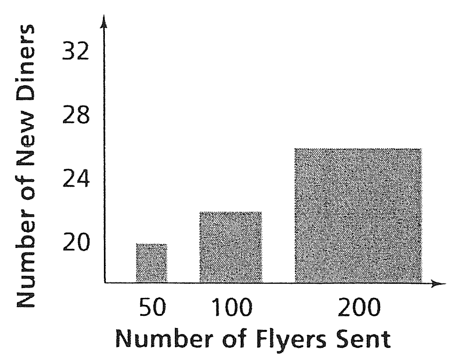
Problem 10.1

A. Use the bar graph below to answer parts (1) – (5).



1. Which test has an average score that appears to be twice as high as Test A?
2. What is the actual difference between Tests A, B, and C?
3. What makes this graph misleading? How should it be corrected?
4. Make a table of tests and average scores.
5. Draw a new bar graph for the data that is not misleading.

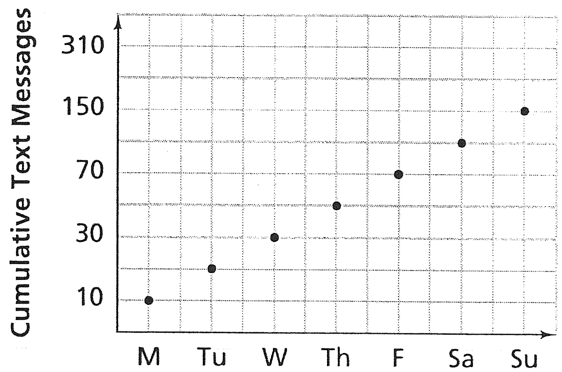
B. Fernando's Grille wants to attract new customers. They hire Alan's Ads to help with the campaign. Alan and his staff pass out flyers one day each week for a month. Alan draws the graph to show the results of the flyer campaign.



1. What appears to happen as the number of flyers sent doubles?
2. What actually happens as the number of flyers sent doubles?
3. Why would Alan want to use this graph?
4. What should be changed to make the graph more accurately reflect the data?

Exercises

1. Madison makes the graph below to show the number of text messages she sends during one week.



   a. Why is Madison's graph misleading?

   b. Why would Madison want to make her graph misleading?

   c. Make a table of days and cumulative number of text messages.

   d. Draw a new graph for the data that is not misleading.

2. Nicolo surveyed his classmates to determine what the new school color should be. The table shows the results.

Color	Number of Students
Blue	8
Green	7
Red	10

   a. How can Nicolo make it seem that blue is twice as popular as green?

   b. How can Nicolo make it seem that blue is four times as popular as green?

Topic 10: Misleading Data Displays

At a Glance

PACING 1 day

Mathematical Goals

- Explain how misleading representations affect interpretations and conclusions about data.

Guided Instruction

To introduce the topic, discuss the characteristics of a bar graph. Ask:

- *Why do you use a bar graph?* (to compare amounts)
- *Why is it easier to compare quantities with a bar graph than in a table?* (Bar graphs are visual and can be interpreted quickly.)
- *What is one of the first things you notice about a bar graph?* (the height of the bars)
- *When one bar represents an amount of 40 and the bar next to it represents an amount of 80, how do you expect to see this reflected in the graph?* (The bar of 80 should be twice as high as the bar of 40.)
- *If one bar is twice as wide as the next bar, what would you think about the data being presented?* (The wider bar has greater importance.)

Once you are satisfied that the students understand bar graphs, move onto the characteristics of a line graph. Ask:

- *Why do you use a line graph?* (to show changes over time)
- *Why is it easier to see changes over time with line graph than in a table?* (Line graphs are visual quickly show upward or downward trends.)

To summarize, ask:

- *How does the data compare in a graph that has been scaled properly with one that has not?* (The data is the same.)
- *Then why is it so important to scale properly?* (The impressions given by the properly-scaled and the improperly-scaled graphs are different.)
- *When would someone choose an inappropriate scale?* (to influence the person using the graph)

ACE Assignment Guide for Topic 10

Core 1–2

Answers to Topic 10

Problem 10.1

A. 1. Test C.

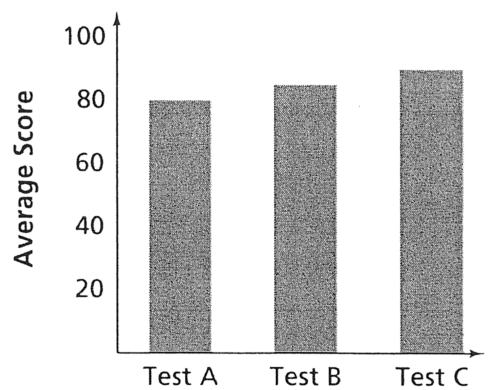
2. A to B: 10, A to C: 15, B to C: 5

3. The values on the y-axis begin at 60, with no indication that the values from 0 to 60 have been skipped.

4.

Test	Average Score
A	75
B	85
C	90

5.



B. 1. The number of new diners appears to quadruple.

2. The number of new diners increases by 2.

3. Alan wants to prove to Fernando's Grille that his flyer campaign is a attracting a lot of new diners.

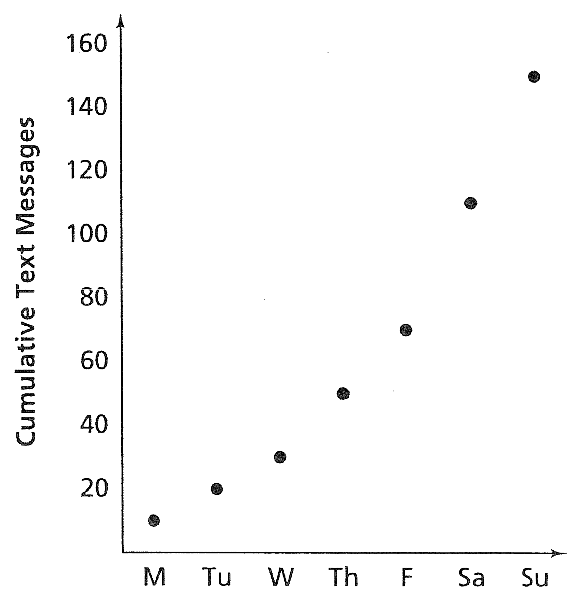
4. The scale on the y-axis needs to begin at 0, and the bars should be the same width.

Exercises

1. a. Madison's graph does not use a consistent scale on the y-axis.

b. Answers may vary. Sample: Madison wants to prove to her parents that the number of text messages is constant rather than increasing per day.

Day	Cumulative Text Messages
M	10
Tu	20
W	30
Th	50
F	70
Sa	110
Su	150



2. a. Nicolo could use a y-axis that begins at 6 and has increments of one vote.

b. Nicolo could use one square unit for green, and then increase each dimension of the square to make four square units for blue.

Name ______________________ Date __________ Class __________

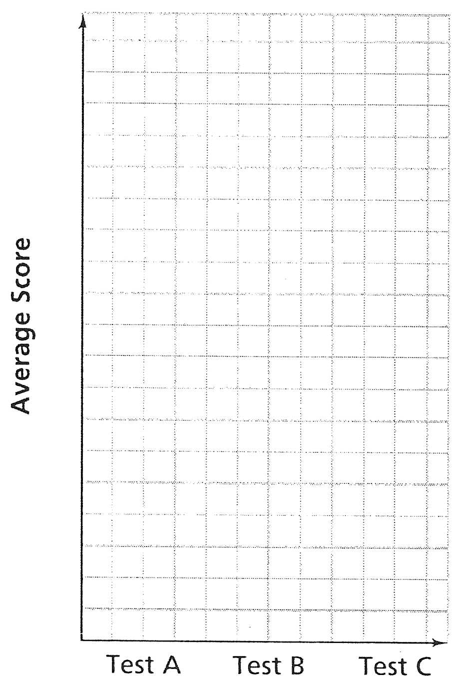
Labsheet 10.1

Topic 10

A. 4.

Test	Average Score
A	
B	
C	

5.



Name ______________________ Date __________ Class __________

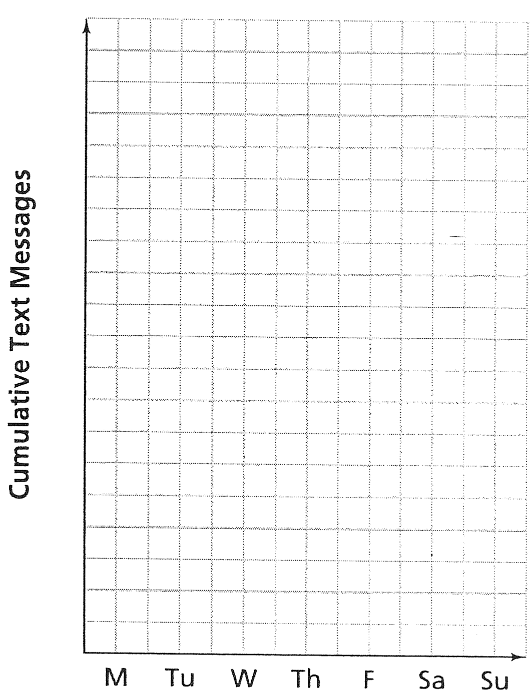
Labsheet 10ACE Exercise 1

Topic 10

1. c.

Day	Cumulative Text Messages
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	

d.



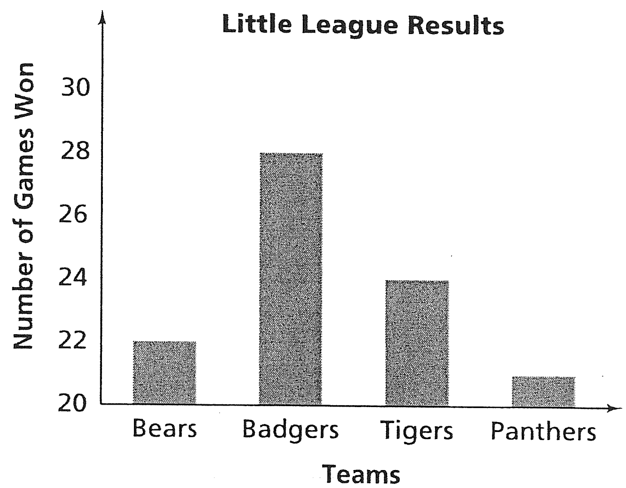
Topic 6: Misleading Graphs

for use before ***Samples and Populations*** **Investigation 1**

When making graphs, choose an appropriate scale divided into equal intervals that fit the data being presented. A scale that is too widespread or too narrow can give the reader a false visual impression. Graphs that use those techniques are **misleading graphs.**

Problem 6.1

A. Use the bar graph below to answer parts (1)–(6).



1. Which team appears to have won twice as many games as the Panthers have won?
2. Find the difference in the number of wins between the two teams of part (1).
3. What do you notice about the vertical scale of the bar graph?
4. Make a table of teams and wins.
5. Draw a bar graph from your table of values using a vertical scale starting at a zero.
6. Which graph better represents the data? Explain.

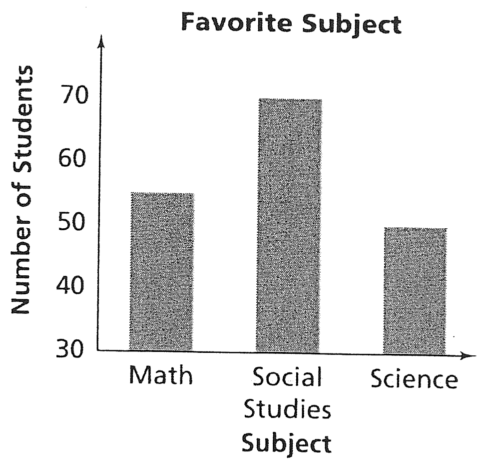
B. A **break symbol** is an insertion at the beginning of a scale to indicate that the scale does not begin at zero.

1. How does a break symbol help you draw a graph?
2. Can a graph with a break symbol be misleading?

C. Kevin states that a graph is misleading if the scale does not begin at zero. Do you agree or disagree? Explain.

Exercises

1. The bar graph shows the results of a survey taken at Brookfield Middle School.



a. Which subject appears to be twice as popular as science?

b. How is the graph misleading?

c. How can you change the graph so that it is not misleading?

2. Use the table below to answer parts (a)–(d).

Company Growth Since Onset of TV Advertisement						
Month	Jan.	Feb.	Mar.	Apr.	May.	June.
Number of New Clients	35	38	40	42	43	44

a. Make a line graph giving the impression that there have been many new clients since the television ad first aired.

b. Make a line graph that indicates a small increase in the number of clients.

c. Which graph more fairly represents the data?

d. Why would someone want to exaggerate the number of new clients?

Topic 6: Misleading Graphs

At a Glance

PACING 1 day

Mathematical Goals

- Explain how misleading representations affect interpretations and conclusions about data.

Guided Instruction

To introduce the topic, discuss the characteristics of a bar graph. Ask:

- *Why do you use a bar graph?* (to compare amounts)
- *What information is placed on the horizontal axis?* (the categories)
- *What information is placed on the vertical axis?* (the amounts)
- *How would you describe the intervals on the vertical axis?* (The intervals are equal in quantity and in height.)
- *Why is it easier to compare quantities with a bar graph than in a table?* (Bar graphs are visual and can be interpreted quickly.)

Review the characteristics of a line graph. Ask:

- *Why do you use a line graph?* (to show changes over time)
- *What information is placed on the horizontal axis?* (measures of time)
- *What information is placed on the vertical axis?* (the amounts)
- *How would you describe the intervals on the vertical axis?* (The intervals are equal in quantity and in height.)
- *Why is it easier to see changes over time with a line graph than in a table?* (Line graphs are visual and quickly show upward or downward trends.)

Summarize by asking:

- *How does the data compare in a graph that has been scaled properly with one that has been scaled improperly?* (The data is the same.)
- *Then why is it so important to scale properly?* (The impressions given by the properly-scaled and the improperly-scaled graphs are entirely different.)
- *When would someone choose an inappropriate scale?* (to influence the person using the graph)

You will find additional work on graphing data in the Grade 7 unit *Data Distributions*.

Vocabulary

- misleading graphs
- break symbol

Materials

- Labsheet 6.1
- Labsheet 6ACE Exercise 2

ACE Assignment Guide for Topic 6

Core 1–2

Answers to Topic 6

Problem 6.1

A. 1. the Bears

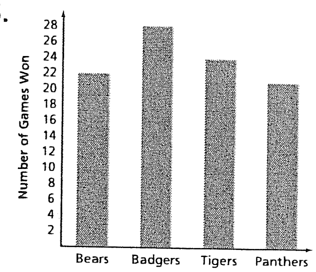
2. 1

3. The scale goes from 20–30; it does not start at zero.

4.

Team	Wins
Bears	22
Badgers	28
Tigers	24
Panthers	21

5.



6. The graph of part (5) better represents the data because the height of each bar is appropriate for its value. A visual comparison of the teams is accurate and not distorted.

B. 1. You can skip small numbers when the data represents large numbers.

2. When there is a break in the graph, the area of the bars is not proportional to the values.

C. Answers may vary. Sample: A graph is misleading when a scale does not begin at zero.

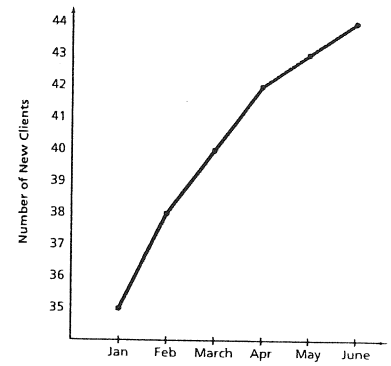
Exercises

1. a. social studies

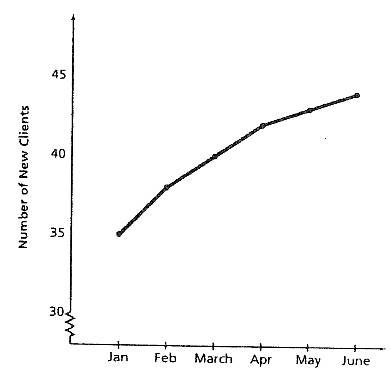
b. The vertical scale starts at 30.

c. Change the vertical scale to start at zero.

2. a. Answers may vary. Sample:



b. Answers may vary. Sample:



c. the second graph

d. Answers may vary. Sample: A salesman might want to show that he is doing a good job by showing a large increase in new clients.

Name ______________________ Date ____________ Class ____________

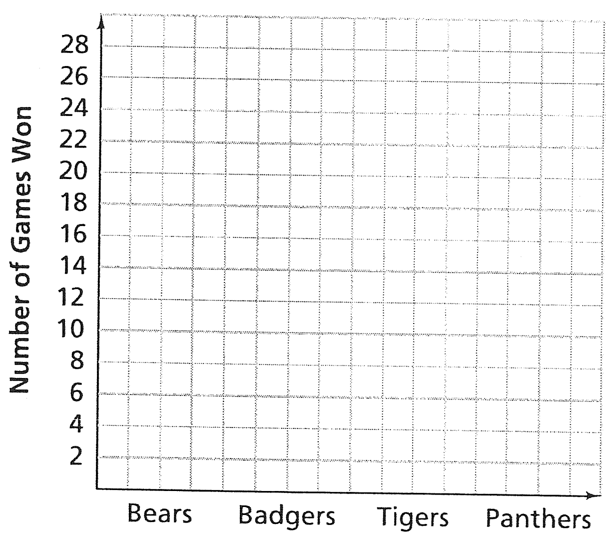
Labsheet 6.1

Topic 6

A. 4.

Team	Wins
Bears	
Badgers	
Tigers	
Panthers	

5.

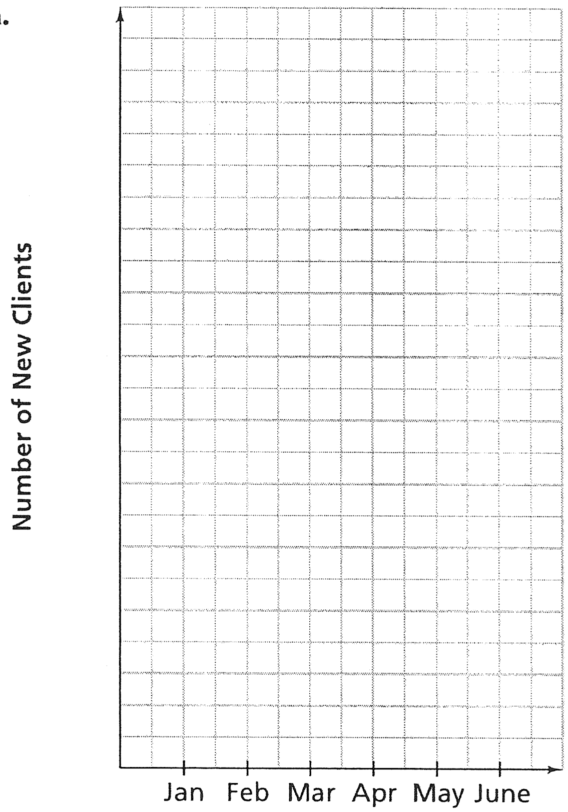


Name ______________________ Date ____________ Class ____________

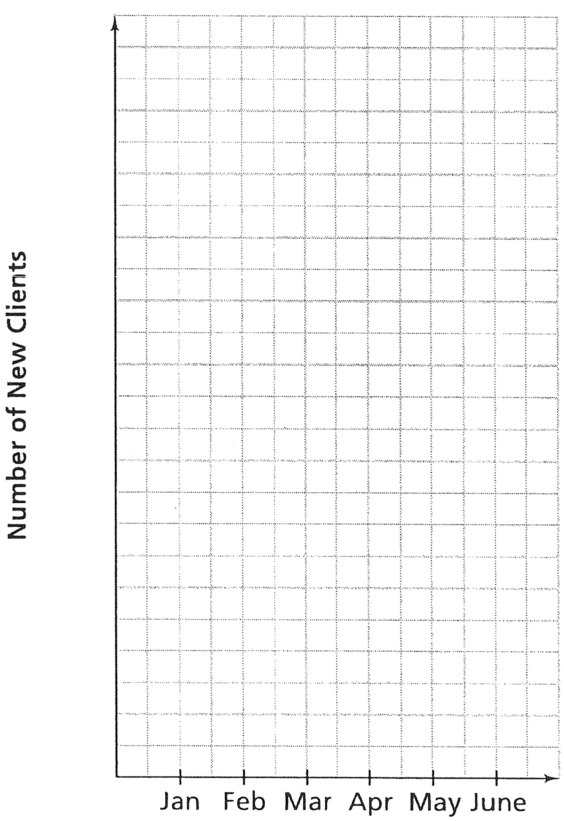
Labsheet 6ACE Exercise 2

Topic 6

2. a.



b.



Additional Practice Pages

Name ______________________ Date ____________ Class ____________

Skill: Mean, Median, and Mode

Investigation 3

Data About Us

For Exercises 1–3, use the table.

1. What is the mean height of the active volcanoes listed to the nearest foot?

2. What is the median height of the active volcanoes listed?

3. What is the mode of the heights of the active volcanoes listed?

Active Volcanoes	
Name	**Height Above Sea Level (ft)**
Cameroon Mt.	13,354
Mount Erebus	12,450
Asama	8,300
Gerde	9,705
Sarychev	5,115
Ometepe	5,106
Fogo	9,300
Mt. Hood	11,245
Lascar	19,652

The sum of the heights of all the students in a class is 1,472 in.

4. The mean height is 5 ft 4 in. How many students are in the class? (1 ft = 12 in.)

5. The median height is 5 ft 2 in. How many students are 5 ft 2 in. or taller? How many are shorter?

The number of pages read (to the nearest multiple of 50) by the students in history class last week are shown in the tally table.

Pages	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750
Tally	I		II	~~IIII~~ I	I	~~IIII~~	III	IIII	I	I					I

6. Find the mean, the median, and the mode of the data.

7. Are there any outliers in this set of data.

8. Do any outliers raise or lower the mean?

9. Would you use the mean, median, or mode to most accurately reflect the typical number of pages read by a student? Explain.

Name ____ Date ____________ Class ____________

Skill: Stem-and-Leaf Plots

Investigation 2

Data About Us

The stem-and-leaf plot at the right shows the number of baskets scored by one of ten intramural teams last season.

5	2 6 9
6	0 4 6
7	1 5
8	4 8

Key: 8 | 4 means 84

1. How many data items are there?

2. What is the least measurement given?

3. What is the greatest measurement given?

4. In how many games did the team score less than 70 baskets?

For Exercises 5–11, use the stem-and-leaf plot below.

Ages of Grandparents

stem	leaf
6	7 8 8
7	0 1 2 3 4 9 9
8	1 3 3 3 4 7
9	0 2 5

Key: 7 | 0 means 70

5. What is the age of the youngest grandparent?

6. What is the age of the oldest grandparent?

7. How many grandparents are 79 years old?

8. How many grandparents are older than 74?

9. How does the data vary?

10. What is the median?

11. What is the mode?

Name ______________________ Date ____________ Class ____________

Additional Practice

Investigation 2

Data About Us

1. The members of the drama club sold candy bars to help raise money for the school's next play. The stem-and-leaf plot below shows how many candy bars each member of the drama club sold.

Candy Bars Sold by Drama Club

1	0 1 1 1 2 3 5 6 9
2	1 1 1 1 4 7
3	2 3 4 8
4	1 4 9
5	2 3 5 5 8

Key: 3 | 2 means 32 candy bars

a. How many students are in the drama club?

b. How many students sold 25 or more candy bars?

c. How do the numbers of candy bars sold by each student vary?

d. What is the typical number of candy bars sold by each student?

2. Earl rolls 6 six-sided number cubes and finds the sum of the numbers rolled.

a. What are the least and greatest sums Earl can roll? Explain.

b. What do your answers for part (a) tell you about the sums Earl can roll?

c. Earl rolled the number cubes several times and recorded each sum. Here are Earl's results:

27, 21, 17, 18, 21, 18, 25, 32, 8, 19, 21, 20, 26, 21, 11, 23, 33, 19, 9, 12, 17

Make a stem-and-leaf plot to display Earl's data.

d. Using your stem plot, find the typical sum rolled. Use the median and range to explain your reasoning.

Name ______________________ Date ____________ Class ____________

Skill: Line Plots

Investigation 1

Data About Us

Ms. Makita made a line plot to show the scores her students got on a test. At the right is Ms. Makita's line plot.

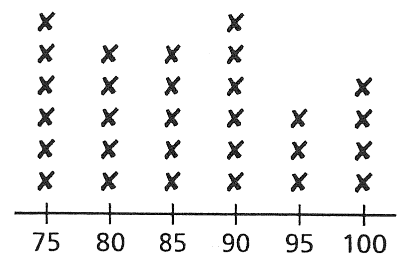
1. What does each data item or ✗ represent?

2. How many more students scored 75 than scored 95?

3. How many students scored over 85?

4. What scores did the same number of students get?

Test Scores



For Exercises 5–8, use the line plot at the right.

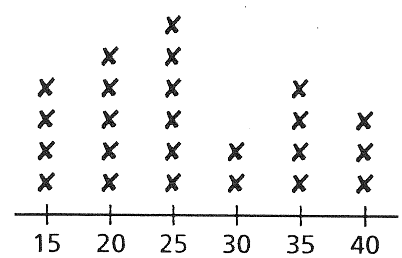
5. What information is displayed in the line plot?

6. How many students spent time doing homework last night?

7. How many students spent at least half an hour on homework?

8. How did the time spent on homework last night vary?

Time Spent Doing Homework Last Night (min)



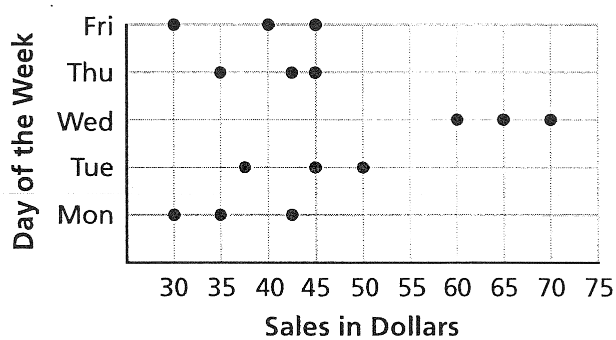
9. A kennel is boarding dogs that weigh the following amounts (in pounds).

5	62	43	48	12	17	29	74
8	15	4	11	15	26	63	

a. How do the dogs' weights vary?

b. How many of the dogs weigh under 50 pounds?

3. Taryn and Travis work in the student store at their school. They made the coordinate graph below to show the total sales each day for three weeks. There are three points corresponding to each weekday because Taryn and Travis recorded their data for the three weeks on a one-week graph.



a. What were the total sales on Tuesdays for the three weeks Taryn and Travis collected their data?

b. Which day of the week seems to be the best for sales at the student store? Explain your reasoning.

c. Which day of the week varies the most for total sales? Explain.

d. How do the sales for the entire three-week period vary?

e. What is the median of the total sales for Fridays? What is the median of the total sales for the three weeks Taryn and Travis collected data?

f. Describe the pattern of sales during a typical week at the student store.

4. Emily rolled two four-sided number cubes 12 times and computed the sum for each roll. She recorded the results as ordered pairs. The first coordinate is the number of the roll, and the second coordinate is the sum for that roll. For example, (9, 2) means that on her ninth roll Emily rolled a sum of two. The results of Emily's rolls were: (1, 7), (2, 8), (3, 3), (4, 4), (5, 6), (6, 3), (7, 5), (8, 6), (9, 2), (10, 4), (11, 5), (12, 5).
   a. Make a coordinate graph of Emily's data. Use the horizontal axis for the number of the roll and the vertical axis for the result.
   b. What is the mode of the sums of Emily's rolls? Explain.
   c. How do the sums vary?
   d. What is the median of the sums? Explain.
   e. Does the coordinate graph you made in part (a) show a pattern in Emily's number-cube rolls? Explain.

For Exercises 5–7, use the stem-and-leaf plot below.

Students' Foot Lengths

1	7
2	0 0 0 0 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 4 4 4 5 5 6 7 7 8
3	0 2

5. How many students are in the class?
   A. 3 B. 12 C. 30 D. 33
6. How do the foot lengths for this class vary?
   F. 1 to 3 G. 7 to 2 H. 17 to 32 J. 20 to 28
7. What is the median foot length for this class?
   A. 2 B. 20 C. 22 D. 24.5

Name ______________________________________ Date ___________ Class ___________

Skill: Histograms

Investigation 1

Samples and Populations

1. Would the data below be better displayed on a histogram with 3-minute intervals or 5-minute intervals? Explain.

Time to Walk to School															
Time (min.)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tally			II	IIII	III	IIII	I	II			I	I			I

2. Make a histogram for the time it takes the group of students in Exercise 1 to walk to school.

3. Make a histogram for the data. Use the intervals in the table.

Hours Spent Doing Homework

Number of Hours	Frequency
1 – 1.75	1
2 – 2.75	1
3 – 3.75	2
4 – 4.75	6
5 – 5.75	8
6 – 6.75	3
7 – 7.75	2
8 – 8.75	1

What Do You Expect?

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations /Additional Targets
Problem 1.1, What Is In the Bucket? P. 5 *(Use vocab of "Complementary Event" throughout unit)*	1		Should Do	**7.4.A Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.** **7.4.B Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.** **Additional Targets:** • **I can determine the probability of an event and represent it as a fraction, decimal, or percent.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Problem 1.2, Matching Colors, p. 6	1		Should Do	
Problem 1.3, Making Purple, p. 8	1		Should Do	
Problem 1.4, Making Counting Trees, p. 10	1		Must Do	
ACE #9-13 pp16-17 ACE #18 p29 and Mathematical Reflections p. 21 #3	1			
CMP2 Problem 2.1 Making Purple Game/ Area Models and Probability p20-23	1	Binder/CMP2(7 disc)		
Problem 2.1, Playing the Addition Game, p. 22 (use area model strategy)	1		Must Do	
Problem 2.2, Playing the Multiplication Game, p. 23 (use area model strategy)	1		#1 Must Do	
CMP2 Problem 2.2 Choosing Paths/Using an Area Model p. 24-25 ACE #6, 7 p. 30	1	Binder/CMP2 (7 disc)		
ACE, Investigation 2, #10, - 13, and 20, p. 27 – 30, and Additional Practice, Investigation 1, #4, p. 164	1			
Mathematical Reflections, Investigation 2, p. 31	1			
What Do You Expect? Unit Assessment, Use Check-Up in place of Unit Assessment	2			
Review and Reflect Common Assessment Student Self-Assessment	1			
Total Instructional Days for What Do You Expect?	**14**			

Contents in What Do You Expect?

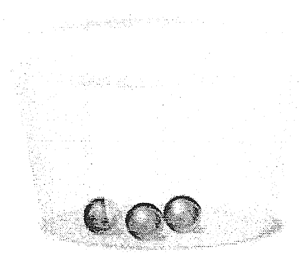
Investigation 2

Analyzing Situations Using an Area Model

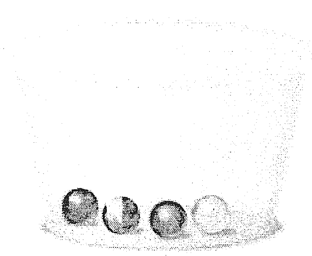
Each turn in the games of chance in the last Investigation involved two actions. For example, in the spinner game, you spun the pointer twice and then determined the outcome. You determined the theoretical probabilities of these games using a variety of strategies.

In this Investigation, you will learn how to use an area model to analyze probability situations that involve more than one action on a turn. You can analyze games such as the Red and Blue game in Problem 1.2 using an **area model.**

Bucket 1 contains three marbles—one red and two greens. Bucket 2 contains four marbles—one red, one blue, one green, and one yellow.

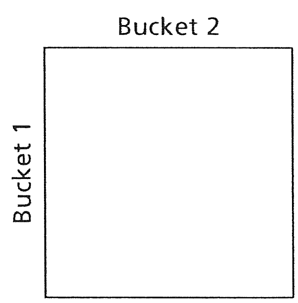


Bucket 1

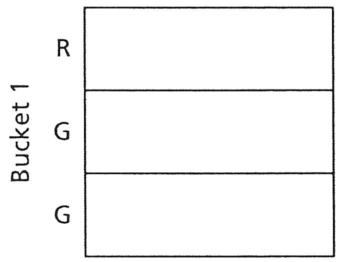


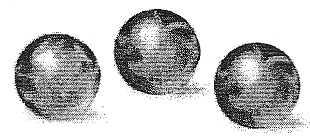
Bucket 2

- Draw a square on grid paper. Suppose the square has an area of 1 square unit. We use the square to represent a probability of 1.

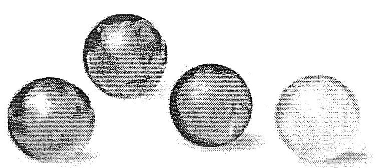


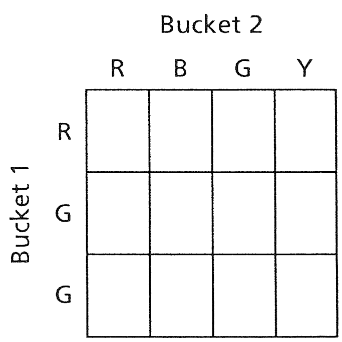
- The first bucket has three equally likely choices: red, green, and the other green. Divide the square into three sections with equal areas. The areas of the sections represent the probabilities of the three choices. Label the sections.





- For the second action of choosing a marble from Bucket 2, subdivide the diagram to represent the probabilities of the equally likely choices: red, blue, green, and yellow. Label these new Bucket 2 sections.





- Each subregion formed represents one of the outcomes: RR, RB, RG, RY, GR, GB, GG, and GY.

Bucket 2

Bucket 1	R	B	G	Y
R	RR	RB	RG	RY
G	GR	GB	GG	GY
G	GR	GB	GG	GY

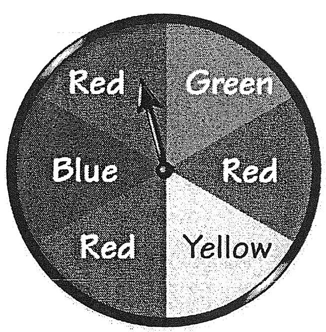
- The area of each subregion represents the probability for each outcome.

What is the probability of choosing an RR? RB? RG? RY? GR? GB? GG? GY? YY?

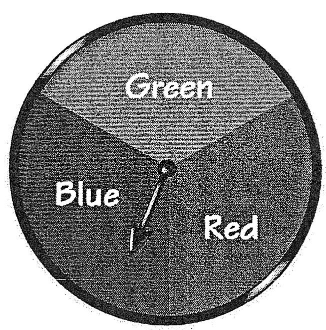
What is the probability of choosing a red from either bucket?

2.1 Making Purple

A popular game at a school carnival is a spinner game called Making Purple. To play the game, a player spins the pointer of each spinner below once. Suppose a player gets red on one spinner and blue on the other spinner. The player wins, because red and blue together make purple.



Spinner A



Spinner B

Problem 2.1 Area Models and Probability

A. Play the Making Purple game several times. Record the results of each turn. Based on your results, what is the experimental probability that a player will "make purple" on a turn?

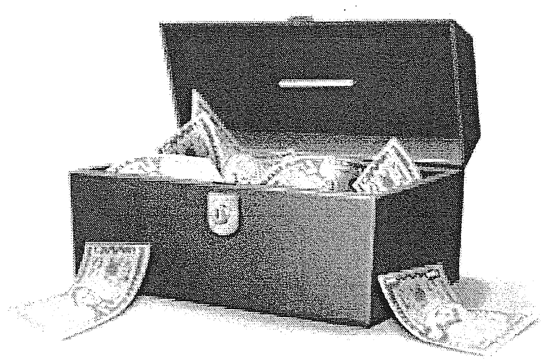
B. Use an area model to determine the theoretical probability that a player will make purple on a turn.

C. How does the experimental probability of making purple compare with the theoretical probability of making purple?

D. The cost to play the game is $2. The winner gets $6 for making purple. Suppose 36 people play the game.

1. How much money will the school take in from this game?
2. How much money do you expect the school to pay out in prizes?
3. How much profit do you expect the school to make from this game?

ACE Homework starts on page 27.



2.1 Making Purple

Goal

- Use an area model to analyze the theoretical probabilities for two-stage outcomes

This problem involves spinning two spinners that are subdivided into sections labeled with different color names. If one spinner comes up red and the other blue, then the color purple is made and the player is a winner. An area model is used to analyze the theoretical probability for the situation.

Launch 2.1

Demonstrate how to analyze a two-stage outcome using an area model. The student edition uses Problem 1.2 to demonstrate the method.

Alternatively, you could first play the Making Purple game to determine experimental probabilities and then, with the whole class, demonstrate how to determine theoretical probabilities using an area model.

Describe the Making Purple game to the class. Demonstrate one or two turns on the two spinners using Transparency 2.1B, if possible.

Suggested Question Ask:

- *Do you think purple and not purple are equally likely outcomes?*

Most students will intuitively know that the two outcomes are not equally likely but they may not be sure how to find the probability of making purple. Some may think that the probability of making purple is $\frac{1}{3} + \frac{1}{6}$ because one spinner is divided into thirds and the other into sixths. Others may offer other suggestions. Again, don't confirm or refute their conjectures. These ideas will be revisited in the summary.

Let students work in pairs.

Explore 2.1

Circulate as pairs work, assisting those who are having trouble analyzing this two-stage game. The problem asks many questions similar to those asked in Investigation 1. Revisiting these questions gives you a chance to help students who are still struggling with these ideas.

Suggested Questions Some students may need help in labeling and interpreting the area model. Rather than show them again, ask them some questions.

- *For spinner A, what are the probabilities of getting each color?* ($\frac{1}{6}$ for blue, yellow, and green and $\frac{3}{6}$ for red.)
- *How can you represent this on the square?* (Divide the square into six equal regions.)
- *Label each of the regions green, blue, red, or yellow.*

Note that students can subdivide the square starting from the left side or the top side. It makes no difference, but for ease in comparing student work in the summary, encourage the students to start with the left side of the square.

- *For spinner B, what are the probabilities of getting each color?* ($\frac{1}{3}$)
- *How can you represent this on the square?* (Subdivide the square into three equal parts starting with the top side of the square.)
- *How many regions do you have?* (18)
- *What does each region represent?* (The results of two spins.)
- *Label each region with the outcome it represents.* (RB, RY, etc.)
- *Which regions represent purple?* (Those labeled RB or BR.)
- *What is the probability of getting purple?*
- *What are some other probabilities for this game?*

Going Further

For those who finish early, ask:

- *What if you have a choice of spinning each spinner once or of spinning one spinner twice? Is the probability of getting purple still the same? Explain.*

Summarize 2.1

Begin with Question A. Elicit students' strategies for finding the experimental probabilities for making purple. To find the experimental probabilities, students analyze their data, count the number of times purple was made, and write a ratio that compares this amount to the total number of trials.

Before discussing the theoretical probabilities, ask what other strategies could be used to determine the outcomes. Some students may try to use a list. Some may try a tree.

Call on one or two students to demonstrate how they found the theoretical probabilities. Compare these with the experimental probabilities.

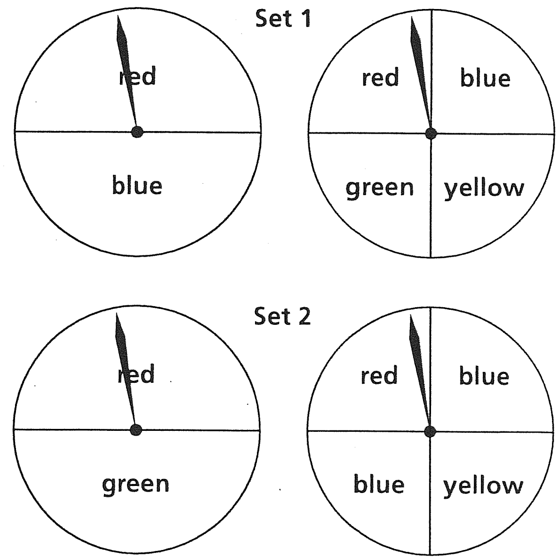
Discuss Question D. Suggest a number of players other than 36. Ask the class to answer the questions in Question D using this number. Or, have each group pick a different number and then answer the questions.

- *Compare your answers using this new number of players to the answers you got for 36 players.*

For example, suppose 72 players played the game. The school still makes a profit, but it is larger, \$48 rather than \$24. The school can expect to make about \$0.67 on average for each player. This is the same no matter how many players play the game.

Check for Understanding

You could display two sets of spinners like those below and ask students to decide which set is more likely to make purple. In this instance, the two sets have the same probability of making purple, namely $\frac{1}{4}$.



2.1 Making Purple

At a Glance

PACING 1 day

Mathematical Goal

- Use an area model to analyze the theoretical probabilities for two-stage outcomes

Launch

Demonstrate how to analyze a two-stage outcome using an area model. Alternatively, play the Making Purple game to determine experimental probabilities and then demonstrate how to determine theoretical probabilities using the area model.

Describe the Making Purple game to the class. Demonstrate one or two turns on the two spinners. Ask:

- *Do you think* purple *and* not purple *are equally likely outcomes?*

Let students work in pairs.

Materials

- Transparency 2.1A
- Transparency 2.1B

Vocabulary

- area model

Explore

As pairs work, assist those who are having trouble analyzing this two-stage game.

Some students may need help in labeling and interpreting the area model. Ask questions about the model.

- *For spinner A, what are the probabilities of getting each color?*
- *How can you represent this on the square?*

As a challenge for those who finish early, consider the following question.

- *What if you have a choice of spinning each spinner once or of spinning one spinner twice? Is the probability of getting purple still the same? Explain.*

Materials

- Labsheet 2.1
- Bobby pins or paper clips

Summarize

Begin with Question A. Elicit students' strategies for finding the experimental probabilities for making purple.

Before discussing the theoretical probabilities, ask what other strategies could be used to determine the outcomes. Compare the theoretical probabilities to the experimental probabilities.

Discuss Question D. Suggest a number of players other than 36. Ask the class to answer the questions in Question D using this number.

- *Compare your answers using this new number of players to the answers you got for 36 players.*

Materials

- Student notebooks

continued on next page

Summarize *continued*

Check for Understanding

You could display two sets of spinners like those on page 39 of the Teacher's Guide and ask students to decide which set is more likely to make purple. In this instance, the two sets have the same probability of making purple, namely $\frac{1}{4}$.

ACE Assignment Guide for Problem 2.1

Differentiated Instruction Solutions for All Learners

Core 2, 3

Other *Applications* 1; *Connections* 13, 14; *Extensions* 25

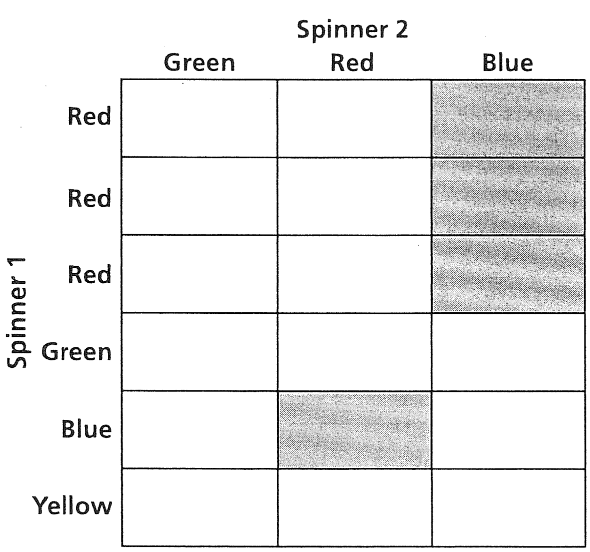
Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units Exercise 13: *Data About Us*; Exercise 14: *Accentuate the Negative*

Answers to Problem 2.1

A. Answers will vary. Some may be close to $\frac{4}{18}$ or $\frac{2}{9}$.

B. The diagram below is a correct area model. The shaded portions represent the ways to get purple with the two spinners. $P(\text{purple}) = \frac{4}{18}$.

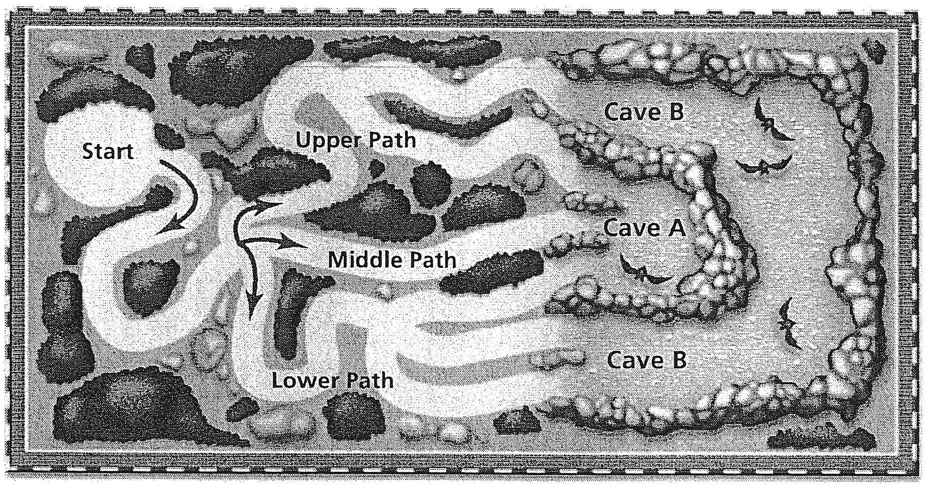


C. They are not necessarily the same. However, they may be close to each other due to the number of times the game was played.

D.
1. The school will take in \$72 from this game.
2. The school would expect to pay \$48 for prizes.
3. The school would expect to make \$24 from this game.

2.2 Choosing Paths

Kenisha is designing a game involving paths through the woods that lead to caves. Before the game is played the player chooses either Cave A or Cave B. Next, the player starts at the beginning and chooses a path at random at each fork. If the player lands in the cave that was chosen in the beginning, he or she wins a prize.



Getting Ready for Problem 2.2

- Are you more likely to end in Cave A or in Cave B? Why?

The 18 students in Sarah's class design an experiment to find the likelihood of ending in Cave A or in Cave B. For each trial, they trace the path beginning at Start, and use a number cube to make the choice of direction whenever there is a split in the path.

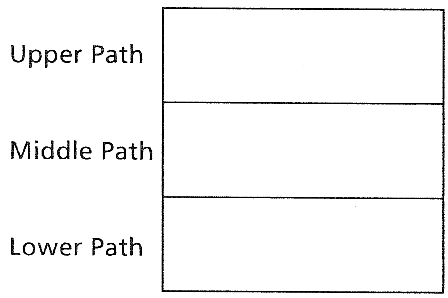
- Is this a good way to find the experimental probability of the game? Explain.
- Are there other ways to make choices at a split in the path? Explain.

Problem 2.2 Using an Area Model

A. Carry out the experiment to simulate the 18 students playing the game and note the cave where each student ends.

B. What is the experimental probability of landing in Cave A? Of landing in Cave B?

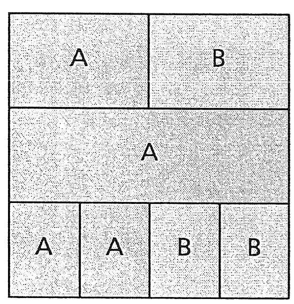
C. Miguel draws this diagram to help him find the theoretical probabilities of ending in Cave A or in Cave B.



1. Explain what Miguel has done so far. Does this look reasonable?
2. Complete an area model to find the theoretical probabilities of ending in Cave A or Cave B. Show your work.

D. How are your experimental probabilities from Question A related to the theoretical probabilities?

E. Kenisha designs a new version of the game. It has a different arrangement of paths leading to Caves A and B. She makes the area model below to analyze the probabilities of ending in each cave.

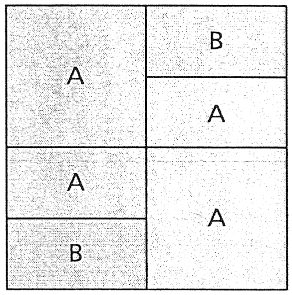


1. Create a path game that fits the model.
2. Find the probability for each outcome.

ACE Homework starts on page 27.

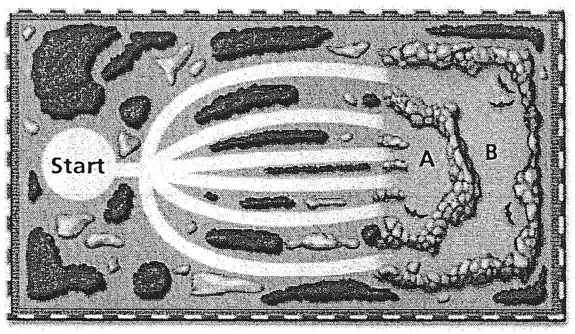
6. Kenisha designs another version of the game in Problem 2.2. The new version has a different arrangement of paths leading into Caves A and B. She makes an area model to analyze the probabilities of landing in each cave.

For Kenisha's new version, what is the probability that a player will end in Cave A? In Cave B?

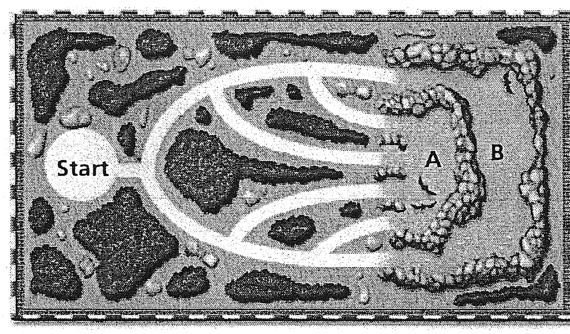


7. **Multiple Choice** Choose the map that the area model in Exercise 6 could represent.

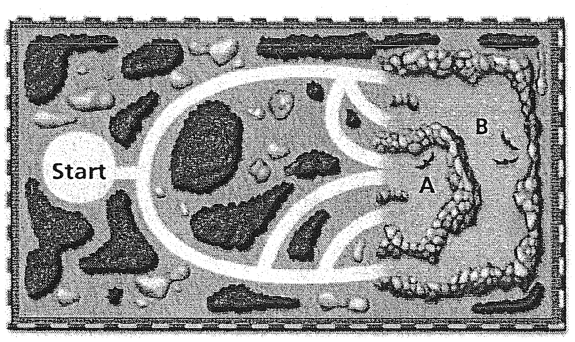
A.



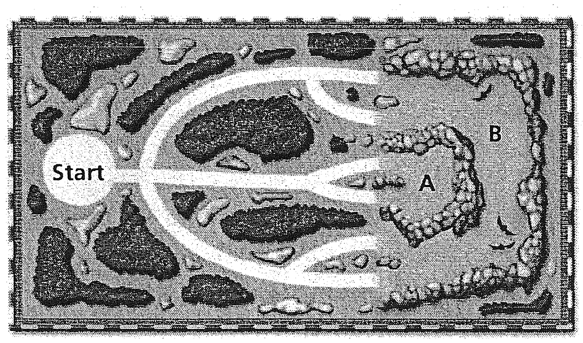
B.



C.



D.



Investigation 2

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 2.1

Core 2, 3

Other *Applications* 1; *Connections* 13, 14; *Extensions* 25

Problem 2.2

Core 4–7

Other *Connections* 15–22, *Extensions* 26; unassigned choices from previous problems

Problem 2.3

Core 8–11

Other *Applications* 12; *Connections* 23, 24; *Extensions* 27, 28; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 23 and other ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 13, 21: *Data About Us*; 14: *Accentuate the Negative*

Applications

1. Deion should score 7 (or 14) points when the spinners make purple and Bonita should score 2 (or 4, if Deion scores 14) points when the spinners do not make purple. This is because Deion has a $\frac{4}{18} = \frac{2}{9}$ chance of making purple and Bonita has a $\frac{14}{18} = \frac{7}{9}$ chance of not making purple. For example, if the spinners were spun 54 times, Deion would get purple 12 times, scoring 12×7 points in the long run, which equals 84 points; Bonita would not get purple 42 times, scoring 42×2 points in the long run, which equals 84 points.

2. **a.** one red, one white, and one blue

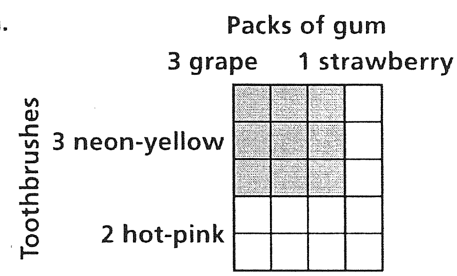
   b. one red, one white, one blue and one green

   c.

	Red	White	Blue	Green
Red			(shaded)	
White				
Blue	(shaded)			

   d. $\frac{2}{12} = \frac{1}{6}$

3. **a.**



   Kira has a $\frac{9}{20}$ probability of drawing a neon-yellow toothbrush and a pack of grape gum.

   b. Of 100 patients, you could expect about 45 ($\frac{9}{20} \times 100$) to draw the same prizes Kira chose.

4. **a.** Possible answer: At each fork that splits into two trails, if even is rolled, go to the right, and if odd is rolled, go to the left. At the fork that splits into three trails, if you roll a 1 or 2, choose the leftmost path; a 3 or 4, choose the middle path; and a 5 or 6 choose the rightmost path.

   b. Answers will vary.

   c.

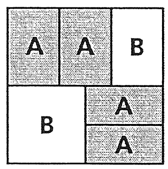
Right Path	Lodge	Lodge	Ski shop
Left Path	Ski lift		Lodge

   For large numbers of experiments, the probabilities should be close to the theoretical probability of $\frac{7}{12}$ for the lodge, $\frac{1}{4}$ for the lift, $\frac{1}{6}$ for the ski shop.

5. a. Cave A: $\frac{7}{12}$; $\frac{1}{6} + \frac{1}{6} + \frac{1}{8} + \frac{1}{8} = \frac{7}{12} \approx 58\%$

Cave B: $\frac{5}{12}$; $\frac{1}{6} + \frac{1}{4} = \frac{5}{12} \approx 42\%$

A square can be divided to show the probability of a player ending in each cave, as shown below.



b. If you played the game 100 times, you could expect to end in

Cave A $100 \times \frac{7}{12}$ = about 58 times and in

Cave B $100 \times \frac{5}{12}$ = about 42 times.

6. Cave A: $\frac{3}{4}$

$\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{3}{4}$

Cave B: $\frac{1}{4}$

$\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

7. B

8. Any of the methods could be used, although some might be easier to use than others.

a. To use a tree diagram, five branches would represent the first draw. From each of these, another five branches would represent the second draw. This would result in 25 different outcomes.

b. To use a list, you have to be careful to distinguish between the different orange and blue marbles (for example, by using symbols such as O1, O2, O3, B1, B2). You would list O1 with each of the other possibilities, and then O2 with each of the other possibilities, and so on.

c. To use an area model, you could divide a square into five rows to represent the first draw. Then, divide each row into five parts to represent the second draw.

d. A chart would resemble an area model, with five rows and five columns, as shown here:

	O	O	O	B	B
O	OO	OO	OO	OB	OB
O	OO	OO	OO	OB	OB
O	OO	OO	OO	OB	OB
B	BO	BO	BO	BB	BB
B	BO	BO	BO	BB	BB

9. Of the 25 possible outcomes, 13 represent marbles of the same color, so you could expect to draw two marbles of the same color, $\frac{13}{25}$ or 26 out of 50 times.

10. Possible answer: Award 12 points for a match; 13 points for a no-match.

11. The best arrangement is one green marble in one container and the remaining marbles in the other container. A square can be divided to show that the probability of drawing green is $\frac{5}{8}$.

12. The best arrangement is one green marble in one container and the remaining marbles in the other container. The probability of choosing green is $\frac{3}{4}$.

Connections

13. a. $\frac{70}{100}$ or 70%

b. $\frac{40}{100}$ or 40%

c. Of the 100 seniors surveyed, 60 drive to school. Of those who don't drive to school, 10 oppose the rule. This is a total of 70, so the probability is $\frac{70}{100}$. (Note: Adding the total number of seniors who drive to school (60) to the total number who oppose the rule (30) is incorrect because it double counts those who drive to school and oppose the rule.) Another way to do this problem is to determine who is not counted, seniors who do not drive to school and who favor the rule, for a total of 30, which leaves 70.

d. One problem with this survey is that it polled only seniors. Because the question concerns a rule that would allow only seniors to drive, many of the other students in the lower grades might oppose it. Thus this survey is probably not a good indicator of the opinions of the entire student body.

14. a.

		Second Spin			
		−1	2	3	−4
First Spin	−1	−2	1	2	−5
	2	1	4	5	−2
	3	2	5	6	−1
	−4	−5	−2	−1	−8

b. Yes, it is a fair game because positive and negative sums are equally likely ($\frac{8}{16}$).

15. $\frac{15}{100}$ 16. $\frac{6}{100}$ 17. $\frac{28}{100}$

18. $\frac{30}{100}$ 19. $\frac{21}{100}$ 20. F

21. Drawings will vary. According to the data, Rich played the game 38 times, and the experimental probabilities that the treasure will be in each room are as follows:

$P(\text{dining room}) = \frac{10}{38} \approx 0.26$

$P(\text{living room}) = \frac{12}{38} \approx 0.32$

$P(\text{library}) = \frac{7}{38} \approx 0.18$

$P(\text{kitchen}) = \frac{4}{38} \approx 0.11$

$P(\text{front hall}) = \frac{5}{38} \approx 0.13$

On a 10 by 10 grid, the dining room should occupy about 26 squares, the living room about 32 squares, the library about 18 squares, the kitchen about 11 squares, and the front hall about 13 squares.

22. a. $\frac{20}{36}$ or $\frac{5}{9}$ b. $\frac{4}{9}$

23. a. $P(\text{landing on A for Dartboard 1}) = \frac{18}{36}$;

$P(\text{landing on A for Dartboard 2}) = \frac{19}{36}$.

Each dartboard can be divided up into 36 equal pieces that are the size of the smallest square piece on each board.

b. i. For each dartboard a person pays \$36 dollars to play 36 times.

For Dartboard 1: A person would make \$0 since the probability of landing on B is $\frac{18}{36}$ and \$2(18) − \$36 = \$0

For Dartboard 2: A person would lose \$2 since the probability of landing on B is $\frac{17}{36}$ or 17 out of 36 times and \$2(17) − \$36 = −\$2.

ii. The carnival would make \$0 from Dartboard 1 and \$2 from Dartboard 2.

c. Assuming the darts land at random, the carnival can expect to make a profit of \$2 for each 36 plays or 5.6 cents per play on this game if players choose Dartboard 2. The carnival can expect to break even on Dartboard 1.

24. a. The factors of 5 are 5 and 1, so there is a $\frac{1}{3}$ chance on each roll of getting a factor of 5.

The probability of getting a factor of 5 on two consecutive rolls is $\frac{1}{9}$. (Note: Students may list all possible combinations, draw an area model, or if they see the connection to multiplying fractions, compute $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.)

b. $\frac{4}{36}$ or $\frac{1}{9}$

c. The answers are the same; rolling the same number cube a second time is equivalent to rolling a second number cube. Each roll of a number cube is independent of other rolls.

Extensions

25. One way to solve this is to make tables, as below. Within each table, each cell represents an equally likely outcome.

		Spinner A			
		Green	Blue	Red	Yellow
Spinner A	Green				
	Blue				
	Red				
	Yellow				

		Spinner B		
		Green	Red	Blue
Spinner B	Green			
	Red			
	Blue			

		Spinner B		
		Green	Red	Blue
Spinner A	Green			
	Blue			
	Yellow			
	Red			

The greatest chance of winning is in spinning spinner B twice. Possible explanation: The charts show that each way to spin the spinners results in two red-blue pairs; the way with the fewest possible outcomes is the best choice.

26. The best arrangement is to put one green marble in the first container, one green in the second container, and the two blue marbles in the other container. The probability of choosing green is $\frac{2}{3}$.

27. Della should put one red marble in one can and the remaining marbles in the other can. This will give her a $\frac{6}{10}$ probability of winning.

28. Answers will vary.

Possible Answers to Mathematical Reflections

1. Students will probably describe some of the situations from this investigation that involved two actions, such as a player choosing paths in the Choosing Paths Game. In this case, the outcomes are: Choosing to go left at the first intersection, then left at the second intersection; choosing to go left at the first intersection, then right at the second intersection, etc.

During class discussion of this question, ask students to think of other situations.

For example, suppose you have three T-shirts: one red, one blue, and one green. You also have four baseball caps: one red and blue, one green and yellow, one red and yellow, and one blue and white. Suppose you first choose a T-shirt at random. Then, you choose a cap at random from those that contain a color that matches the shirt. What is the probability that you will choose each combination with the matching colors?

There are five outcomes. The outcomes and probabilities are described in Question 2.

Another situation is choosing to carry or not to carry an umbrella after listening to the weather forecast.

2. A square area model is appropriate when there are two or more actions in a situation because you can show the different actions. The area of a square represents 1.

The example of selecting a shirt and then a cap from Question 1 is used to show how the area model can be used to determine the probability of selecting a shirt and hat that match. You can use a different example or modify this example.

First, you divide a square into three equal parts to represent the three shirt choices. Next, you need to determine the cap choices for each shirt. For each red shirt, there are two caps (red/blue and red/yellow); for the blue shirt, there are two caps (red/blue and blue/white). Each of these regions is divided in half. The green shirt corresponds to only one cap (green/yellow), so that region is not subdivided. The probability of choosing each cap can be found by adding its fractional areas. The red/blue cap has a probability of $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$, the red/yellow and blue/white caps each have a probability of $\frac{1}{6}$, and the green/yellow cap has a probability of $\frac{1}{3}$.

red shirt	red/blue cap	red/yellow cap
blue shirt	red/blue cap	blue/white cap
green shirt	green/yellow cap	

2.2 Choosing Paths

Goals

- Simulate and analyze probability situations involving two-stage outcomes
- Distinguish between equally likely and non-equally likely outcomes by collecting data and analyzing experimental probabilities
- Use an area model to analyze the theoretical probabilities for two-stage outcomes

Problem 2.2 introduces a new probability context, that of analyzing paths in a game. At various places along the paths, students must choose a path at random until they end up in one of two rooms. Students first simulate the game and assign probabilities using their simulation. An area model is then used to determine the theoretical probabilities.

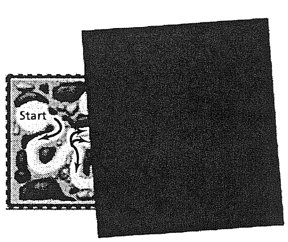
Launch 2.2

Introduce the Choosing Paths game. Display the game screen, which is shown on Transparency 2.2A. Use the Getting Ready to help the class focus on how the game is played and how to use a number cube to help make decisions at each split in the path.

Suggested Questions

- *Are you more likely to end in Cave A or in Cave B? Why?* (Some students may say Cave B because it is larger; some may say the caves have an equal chance of being entered because three paths lead into each room; some may say Cave A because the middle path leads directly into this cave.)

To help students better understand the paths, you could cover up all of the diagram after the first split in the path, as below:



- *Suppose you are playing the game and have to make a random decision at the first split in the path about which part of the path (upper, middle or lower) to follow. How can a number cube help?* (You could take the upper if 1 or 2 is rolled, the middle if 3 or 4 is rolled, and the lower if 5 or 6 is rolled.)
- *Does this give each of the paths the same chance of being chosen?* (Yes.)
- *What is the probability of selecting each one of these three paths?* ($\frac{1}{3}$)

Now move the paper to reveal the upper paths and again ask the question.

- *How can you use the number cube to help you decide which path to take if each is to have an equal chance?* (Let the upper path be selected if a 1, 2, or 3 is rolled and the lower selected if a 4, 5, or 6 is rolled.)
- *What is the probability of selecting one of these paths?* ($\frac{1}{2}$ for each fork of the path)

Now focus on ways to simulate the maze.

- *How can we simulate walking through the maze and choosing paths at random?*
- *Random means that you can't choose paths by picking your favorite number or your favorite direction. At each fork, every path must have exactly the same probability of being chosen. The number cube will be our way to make decisions at random at each split in a path.*
- *Let's play a version of the game in our class. Let's pretend to walk through the maze choosing paths at random.*
- *At each fork or intersection, you must choose a path in such a way that each path has exactly the same probability of being selected.*

Play the game a couple of times or until everyone understands how it is played.

Have number cubes for groups to use to generate random choices.

- *In your group carry out the experiment 18 times and answer Questions A and B. Then, move on to Questions C–E.*

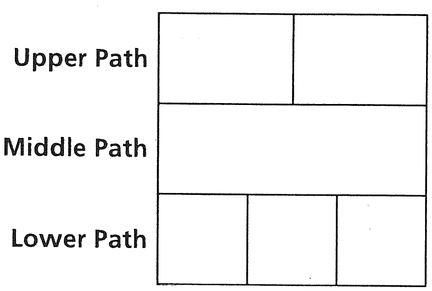
Explore 2.2

Play the game 18 times and record group results.

Suggested Question As groups work, ask questions about what they are discovering.

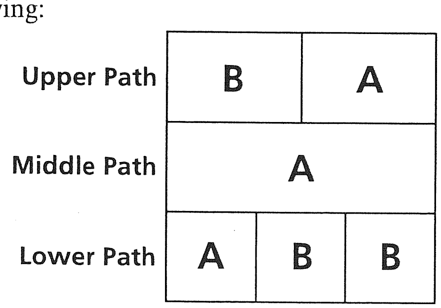
- *Which cave seems to have the greater probability of the player entering it? What makes you think this?*
- *If you come to a fork that splits into three paths, what probability does each path have of being selected?*
- *Suppose your first choice is to take one of three paths, each of which is followed by a choice of two paths. What is the probability that you will take any given second path?*

When students move to the area model analysis to find a theoretical probability model, some students may try to label the top of the diagram as they did in 2.1. Here labels do not work because each path may have different probabilities. The better way is to reallocate the area allotted to a path into as many equal-sized parts as there are forks in the path (2, 0, or 3) as you read from the upper path to middle path to the lower path. At this stage the diagram should look like the following:



- *After the first division, what fraction of the total area of the square does each of the three paths represent?*

Now the students can make a diagram to show where you end for each of the options to get the following:



- *Now that we have divided a square to represent the different paths, what fraction should we assign to each part of the square?*
- *What is the probability of landing in Cave A? In Cave B?*

As students work on Question E, have them put their path game on a large sheet of paper with the outcomes and probabilities for entering each cave.

Summarize 2.2

Collect groups' strategies for walking the maze and making random path choices at each fork. Be sure to have the class confirm that these strategies make sense.

Have students share the experimental probabilities they found for each cave. Discuss reasons for variation in the data. Ask questions about other ways to make random choices at each split in the path. Help the class to pool their experimental data and to calculate the experimental probabilities based on all the groups' trials. Save the experimental data to compare with the theoretical data in Question C.

Ask students how their experimental probability compared to their initial idea about the cave in which they would end.

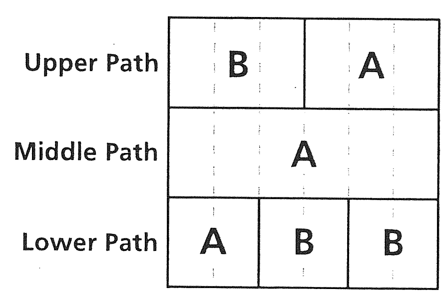
Now move the students to Question C and look at examples of the area diagrams the students complete. Use the diagrams in the Explore as a guide.

- *How did you assign probabilities to ending in Cave A or in Cave B?* (There were three different areas assigned to each cave.

  Cave A: $\frac{1}{6} + \frac{1}{3} + \frac{1}{9} = \frac{11}{18}$

  Cave B: $\frac{1}{6} + \frac{1}{9} + \frac{1}{9} = \frac{7}{18}$

  Some groups will partition the grid into equal-sized parts in order to find the probabilities. Partitioning into 18 parts is shown below.)



Another way to think about this is to write a number sentence for each probability and have the class connect the number sentence back to the area model. For example, the probability of landing in Cave B is: $(\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3})(\frac{1}{3}) + (\frac{1}{3})(\frac{1}{3}) = \frac{1}{6} + \frac{1}{9} + \frac{1}{9} = \frac{7}{18}$.

Choosing a way to simulate random events related to a particular problem is important. Discuss with your class other strategies that could have been used to simulate the path game.

Suggested Questions Ask:

- *Look back at the simulation we used to find the experimental probabilities of ending in Cave A or B. We used a number cube. Could we have used a spinner? If so, how?* (A six-section spinner can work just like a number cube. But we could also use several spinners depending on how many equally likely choices we have to make. In this game we could use two spinners, one with three equal parts and one with two equal parts. Then you just spin the spinner that matches the number of choices.)
- *What are some other ideas about how to simulate the path game?* (Students might suggest colored cubes in buckets, pieces of paper labeled with choices, or other random devices.)

Note: Using the area model provides practice with writing equivalent fractions, adding, and multiplying fractions. Subdividing the square twice is a model for multiplication of fractions that students studied in the sixth-grade unit, *Bits and Pieces II*. This is an opportunity to assess students' facility with fractions.

Mathematics Background

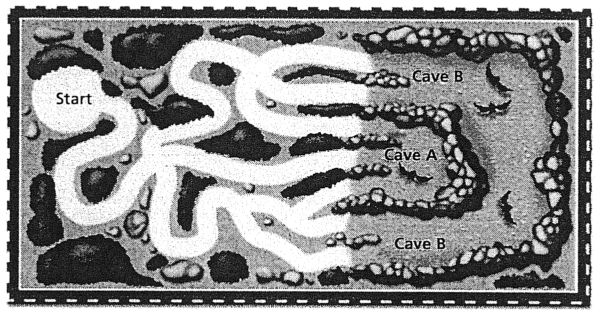
For background on multiplying probabilities, see page 5.

Compare the theoretical and experimental probabilities.

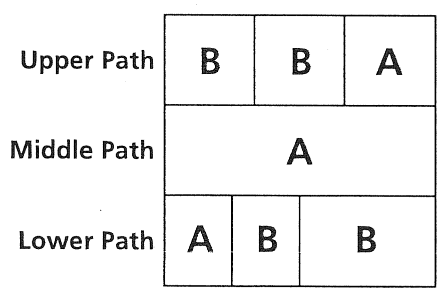
For Question E, you can post the path games that the students created around the cave. You can either have the class explain their paths or have the class move around in groups to check one or two of the paths to see if they match the given analysis. Ask what strategies they used.

Check for Understanding

For a further check on whether students understand how to find the probabilities of successive events, draw another maze for the class to analyze. For example, this maze is shown on Transparency 2.2B.



Give the class some time to think about this example. The associated area model for the example might look as follows:



Adding the fractional parts of the drawing that represent ending in each cave gives

$\frac{1}{9} + \frac{1}{3} + \frac{1}{12} = \frac{19}{36}$ for Cave A and

$\frac{1}{9} + \frac{1}{9} + \frac{1}{12} + \frac{1}{6} = \frac{17}{36}$ for Cave B.

2.2 Choosing Paths

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Simulate and analyze probability situations involving two-stage outcomes
- Distinguish between equally likely and non–equally likely outcomes by collecting data and analyzing experimental probabilities
- Use an area model to analyze the theoretical probabilities for two-stage outcomes

Launch

Display the Choosing Paths game screen shown on Transparency 2.2A. Use the Getting Ready to focus on simulating the maze using a number cube and to ask which cave one is more likely to end in.

- *How could we use a number cube to simulate walking through the maze and choosing paths at random to find an experimental probability?*

Help students see that rolling a 1 or 2 can represent choosing the upper path, a 3 or 4 the middle path, and a 5 or 6 the lower path.

Play a version of the game with the class. As students make their selections, demonstrate them at the board or overhead. To help students focus on the initial path choices, you may want to cover all but the first intersection until they have made their first choice.

You might show the diagram in Question C so that the work in Questions A and B will help them think about Question C.

Point out that Question E asks them to work backwards and create a game board to match the area model of probabilities given.

Have students work in groups of 3 or 4.

Materials

- Transparency 2.2A

Explore

As groups work, ask questions about what they are discovering.

- *Which cave seems to have the greater probability of the player entering it? What makes you think this?*
- *If you come to a fork that splits into three paths, what probability does each path have of being selected?*
- *Suppose your first choice is to take one of three paths, each of which is followed by a choice of two paths. What is the probability that you will take any given second path?*

Materials

- Number cubes
- Colored blocks
- Coins
- Large sheets of paper
- Markers

Summarize

Have students share their experimental probabilities for each cave. Discuss reasons for variation in the data. Ask questions about other ways to make random choices at each split in the path. Pool the class experimental data and compare it with the theoretical data from the area analysis. Have students share their designs for Question E and determine whether they are correct. Ask what strategies they used.

Check for Understanding

For a further check on whether students understand how to find the probabilities of successive events, draw another maze for the class to analyze.

Materials
- Transparency 2.2B
- Student notebooks

ACE Assignment Guide for Problem 2.2

Differentiated Instruction Solutions for All Learners

Core 4–7
Other *Connections* 15–22, *Extensions* 26; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units Exercise 21: *Data About Us*

Answers to Problem 2.2

A. Results will vary.

B. Answers may vary depending on the results in A. Combining all of the class data should bring the experimental probabilities close to the theoretical probabilities of $\frac{11}{18} \approx 61\%$ for Cave A and $\frac{7}{18} \approx 39\%$ for Cave B.

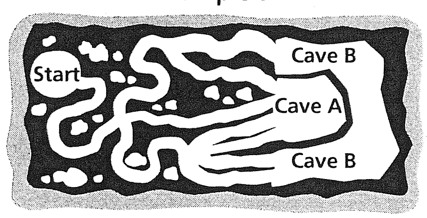
C. See diagrams in the Explore.

1. Miguel has recognized that each of the paths after the first split has a $\frac{1}{3}$ probability of being chosen. He has partitioned the square so that each of the three paths has the same probability.
2. See diagram and discussion in the Explore. $P(A) = \frac{11}{18}$ and $P(B) = \frac{7}{18}$.

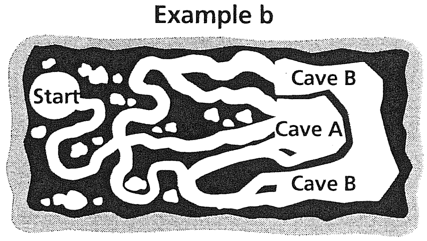
D. If you combined the class's experimental data, their experimental probabilities should be close to the theoretical probabilities.

E. 1. Game screens will vary slightly, but should have a structure similar to the ones shown at right: There are three main paths. The top path splits into two, the middle path does not split, and the bottom path splits into four. Example *c* below is a potential wrong answer (note that the bottom path splits into four paths that are not each equally likely).

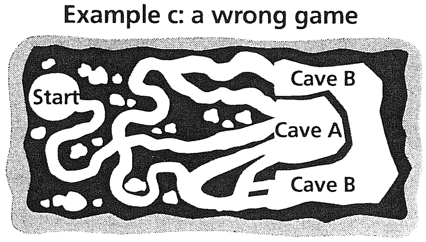
Example a



Example b



Example c: a wrong game



2. Cave A: $\frac{2}{3}$

$(\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3}) + (\frac{1}{3})(\frac{1}{4}) + (\frac{1}{3})(\frac{1}{4}) =$

$\frac{1}{6} + \frac{1}{3} + \frac{1}{12} + \frac{1}{12} = \frac{2}{3}$

Cave B: $\frac{1}{3}$

$(\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3})(\frac{1}{4}) + (\frac{1}{3})(\frac{1}{4}) = \frac{1}{6} + \frac{1}{12} + \frac{1}{12} = \frac{1}{3}$

Additional Practice Pages

Complementary Events: two or more events whose probabilities add up to 1

Mutually Exclusive Events: two events that have no outcomes in common